UNIVERSITY OF JORDAN
FACULTY OF GRADUATE STUDIES
GRADUATE DEPARTMENT OF
INDUSTRIAL ENGINEERING

"CRASHING IN PROGRAM EVALUATION AND REVIEW TECHNIQUE NETWORKS"

By

Moayad Ibrahim Samman

Under the Supervision of

Prof. Arun G. Walvekar

**Submitted in partial fulfillment of the requirements for the Degree of Master of Science in Industrial Engineering - Faculty of Graduate Studies - University of Jordan
Amman - Jordan**

AUGUST 1991

Date of thesis presentation: August 25, 1991

The examining committee considers this thesis satisfactory and acceptable for the Master of Science Degree in Industrial Engineering

Prof. Arun G. Walvekar, Chairman,
Industrial Engineering Department
University of Jordan

Prof. Prakash Mahajan, Member,
Industrial Engineering Department
University of Jordan

Dr. Ghaleb Abbasi, Member,
Industrial Engineering Department
University of Jordan

Dr. Zein Al-Abedeen Tahboub, Member,
Industrial Engineering Department
University of Jordan

Dr. Mohammed Hiyassat, Member,
Civil Engineering Department
University of Jordan

TO MY PARENTS

ACKNOWLEDGMENT

I wish to express deep thanks to Prof. Arun G. Walvekar under whose supervision this thesis was carried out, for his dynamic enthusiasm, unsurpassed guidance and advice on all technical and administrative problems through out this work. Also, I would like to thank Dr. Ghaleb Abbasi and Dr. Mohammed Hiyassat, without whose unlimited generosity in giving help and encouragement this work could not have been accomplished.

ABSTRACT

This thesis aims at producing a mathematical model that is capable of identifying the amount of money that might be invested in certain activities in order to maximize the probability of accomplishing a project and to develop a heuristic to solve the model.

The thesis started with a general introduction afterwhich the objective was identified. This was followed by literature survey discussing the evolution of project management techniques that are pertinent to the subject of the thesis. Risk and uncertainties were also discussed because of their strong relation to the probabilistic nature of the activities. Three exact nonlinear models were developed, because of the combinatorial nature of the activities, large number of iterations will be needed to solve these models. A heuristic was developed, which involved less computations if compared with exact algorithms and was conducive to obtaining a good solution.

To test and support the validity of the heuristic, a theoretical case study was introduced. The results were as expected, however, testing the solution quality was not conducted.

A computer program in Fortran was developed to solve part of the heuristic, while the linear formulation part was solved using the Quantitative Systems for Business (QSB) software package.

ملخص

تهدف هذه الاطروحة الى بناء نموذج رياضي يستطيع تحديد كمية الاموال التي يفضل استثمارها في نشاطات محددة من اجل الحصول على افضل احتمالية لانجاز مشروع ما قبل فترة معينة. اضافة الى ايجاد طريقة استكشافية (Heuristic Method) لحل النموذج الرياضي.

تبدا الاطروحة بمقدمة عامة تستعرض باسلوب مختصر طريقة المسار الحرج واسلوب تقيم ومراجعة الشبكات، يلي ذلك تعريف بالمشكلة التي يطرحها البحث مع بيان الاهداف المتوخاة.

وللتعرف على الاعمال والابحاث السابقة المتعلقة بموضوع الاطروحة تم تخصيص فصل اشتمل على استعراض تاريخي لنشأة وتطور اساليب ادارة المشاريع والمواضيع المرتبطة بموضوع الاطروحة. وفي نهاية هذا الفصل تم وضع الخطوات المتوقع انتهاجها لتحقيق اهداف البحث. نتيجة لطبيعة النشاطات الاحتمالية، تم استعراض المقاييس المختلفة والمتعلقة بتقيم المشاريع تحت ظروف من المخاطرة وعدم التاكد.

تم من خلال الاطروحة تطوير ثلاثة نماذج للمسألة حيث تعطي هذه النماذج الحل الافضل. ولكن نتيجة لطبيعة النشاطات التوافقية والحاجة لعدد كبير من الخطوات التكرارية الحسابية للوصول الى الحل الامثل ، فقد تم تطوير طريقة استكشافية تصل الى حل جيد بعدد اقل من الخطوات مقارنة بالنماذج السابقة .

وللتاكد من صحة وفعالية الطريقة الاستكشافية، فقد تم اختبارها عن طريق مثال نظري حيث كانت النتائج جيدة واظهرت قابلية الطريقة للتطبيق .

للمساعدة في حل الطريقة الاستكشافية فقد تم عمل برنامج على الحاسوب بلغة الفورتران يستطيع حل جزء من الطريقة اضافة الى استخدام برنامج اخر لحل الجزء المتعلق بالصيغ الخطية .

TABLE OF CONTENTS

CHAPTER ONE

INTRODUCTION

Since the introduction of PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method), the use of the network technique as a new management tool has spread rapidly and with far reaching impact. Whether under the name of PERT or CPM the same basic concept has been used for a wide range of planning and control problems.

1.1 SIGNIFICANCE OF PERT

PERT is viewed as a major tool for the management of nonrepetitive or one-time through programs. It represents an important contribution to the family of management science, or operations research techniques used in Project Management [1].

PERT is one of the techniques of management science where it is possible, in the case of actual application, to achieve payoffs with relatively uncertain planning and control problems. The two features of PERT which bring about this advantage are the judicious use of three way estimates, which represent a relatively simple approach to the prediction of time uncertainty and flexibility of updating, which allow a quick reaction to the impact of events as they actually occur.

1.2 SINGLE VERSUS MULTIPLE TIME ESTIMATES

The desirability of the single time estimate compared with three time estimates for each activity has been discussed a great deal. Consideration of the background of different uses, indicates that the desirability of one time estimate compared with three depends upon the environment and the nature of the application [2].

The single time estimate is appropriate in networks used in the process and construction industries. On a construction project, for example, we can expect that the activities or jobs are well known and that past experience provides a basis for a reliable and accurate time estimates. Therefore, using a single time estimate for each activity is a valid and realistic practice [2].

In the defense and space industries, network planning is used primarily in the research and development programs with a large degree of uncertainty. Since many of the activities in such projects have never been carried out before or have been carried out only a few times under very different circumstances, using three time estimates to reflect this uncertainty has definite advantages. The greatest advantage comes in the initial uses of the technique, however; when the planner is sufficiently familiar with the networks, and has gained confidence in using them, he can often use a single

CHAPTER TWO

LITERATURE REVIEW

Both PERT and CPM arrived on the industrial scene at about the same time, and as essentially independent developments. Since the basis work on CPM was done earlier, the subject of CPM will be introduced first.

2.1 CRITICAL PATH METHOD (CPM)

The Critical Path Method (CPM), was developed in 1957 by Morgan R. Walker of the Engineering Service Division of Du Pont and James E. Kelly [1]. Walker and Kelly were concerned with the problem of improving scheduling techniques for such project as the building of a pilot model plant and the shutdown of a plant for overhaul and maintenance. After considering the premise that all the activities of such project must be executed in a well defined sequence, they came up with the arrow diagram as the most logical representation of the interrelationship between jobs for any project. Their arrow diagram and method of calculating the longest or critical path through it were the same as the PERT network and critical path calculations. Kelly and Walker used a single time estimate, and did not go into the problem of uncertainty in time duration of individual jobs [1].

2.2 PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

The Program Evaluation and Review Technique (PERT), was developed in 1958 in the Navy's Special Project Office because of the recognition of Admiral W. F. Raborn [1] that something better was needed in the form of an integrated planning and control system for the Fleet Ballistic Missile (FBM) program, commonly known as the Polaris Weapons Systems [1]. Based upon his support, a research team was established in 1958 to work on a project designated as PERT or Program Evaluation Research Task. By the time of the first Navy report on the subject, PERT has become " Program Evaluation and Review Technique," and thus it has persisted until this day, when it has become part of the everyday language of industry. D. G. Malcolm, J. H. Rosenboom, C.E. Clark, and W. Fazar, all of the original Navy sponsored research team, were the authors of the first publicly published paper on PERT, which was published in the September, 1958, issue of Operations Research [1]. Because of the complexity and size of the Polaris program, this original research team decided to restrict the initial application of PERT to the time area, which, as it turned out was a very wise decision [1].

Since these original contributions to the development and application of the network technique, the amount of literature on CPM and PERT, and the number of management systems derived from them, has increased at an exponential rate. As of

1963,any complete bibliography on PERT and CPM would have numbered entries in excess of a thousand [1]. In addition, a compilation of the code names of the various PERT-type system was made by the U.S.Air Force Systems Command in 1963 [1].

Although, this compilation contained many representation, trend toward duplication and redundancy will remain to be seen. One example of the reversal of this trend was the withdrawal in 1962 by the Air Force of its code name PEP (Program Evaluation Procedure), which was equivalent to PERT [1]. In addition, the establishment by the Federal government in 1962 of a uniform guide on PERT/COST, has undoubtfully helped reduce this trend [1].

2.3 STOCHASTIC ACTIVITY NETWORKS (SANs)

Stochastic activity networks (SANs) are widely used in scheduling and management of large projects. However, the analysis of such networks is greatly complicated by stochastic dependencies among network components that arise, for example, when some activities are common to several paths or when several durations are correlated [3]. Conventional analysis techniques are based on restrictive assumptions about the probability distributions of the activity durations or about the topology of the network [3]; these assumptions generally yield approximation of unknown accuracy. Because of its ability to represent faithfully the dependencies among the components of a stochastic activity network and to yield estimates of desired performance measures with controllable accuracy, Monte Carlo simulation [3] is frequently the method of choice for the analysis of such networks.

In the simulation of SAN, the usual objective is to obtain point and confidence-interval estimators for the mean completion time θ of the network. If the random variable Y denotes the completion time of a given SAN, then direct simulation simply computes the sample mean response $\bar{Y}$ from n independent replications of the network to yield an unbiased estimator of θ with var $(\bar{Y})$ = var (Y)/n. Since the variance of $\bar{Y}$ declines as the inverse of the sample size, a large number of replications will usually be required to achieve acceptable precision [3].

Several variance reduction techniques have been proposed for improving the efficiency of activity network simulations, including conditional Monte Carlo, stratified sampling, antithetic sampling, control variates, and combinations of these techniques [3]. Some of the recent work has led to the conclusion that in comparison to the other commonly used variance reduction techniques, the method of control variates is more easily adopted to a wide variety of network configurations and has greater potential to yield large efficiency increases in general applications [3].

2.4 BASIC COMPARISONS BETWEEN CPM AND PERT TECHNIQUES

The CPM and PERT methods are useful at various stages of project management, from initial planning and analyzing alternative programs to scheduling and controlling activities of a given project. In essence, these tools provide a means of determining [4]:

1. Which of the many activities that comprise a project are critical in their effect on total project time - this determines basically what is called the critical path; and

2. how best to schedule all activities in the project in order to meet a target date at a minimum cost - with or without constraints concerning availability of resources and their leveling.

The mechanics of analysis in CPM and PERT uses primarily a geometric representation called network. A network (or graph) is essentially a flow chart of activities or processes (nodes), of which various pairs are connected by branches showing the precedence relationship between them. One can understand the correlation between a network representation and a project which consists of well-defined set of activities that must be performed in some technological sequence. Most commonly the system leads to a single last activity indicating completion. In CPM and PERT the branches are directed and no loops are permitted. Often another diagraming representation is used where activities are represented on branches and nodes represent events in time [4].

CPM method uses a deterministic approach whereby activity durations are defined in exact time, while PERT allocates a probabilistic (unimodal beta function) distribution for activity durations which therefore become random variables. Thus, CPM is best applied to projects in which activity durations and costs may be estimated accurately, while PERT is best applied to research and development projects in which many of the activities have never been performed before. The PERT method seems to be most concerned with prediction before the start of a job, while CPM, with its slack calculation, is intended as a managerial tool during the course of a job [4].

2.5 COMPARISON OF NETWORKING METHODS

The use of CPM and PERT methods has been widely accepted by many managerial organizations. However, these methods were not able to handle some particularly important and very realistic situations. For example, they do not allow handling of alternatives or conditionalities; they assume that all activities must be completed. They do not permit the presence of loops or feed backs used in repetitive processes or in quality control problems. In PERT, the critical path may have a variance smaller than that of any other path, and inversely a second critical path may have a variance much larger than that of the critical path [5].

In view of these deficiencies, newly developed networking tools are being implemented in some specialized sectors of various industries. They include Decision Critical Path Method (DCPM), Linear Flow Graphs, and stochastic networks illustrated herein by Graphical Evaluation and Review Technique (GERT) [5]. An exhibit comparing these methods as well as CPM and PERT is shown in Table 2.1.

2.6 THE THREE TIME ESTIMATE TECHNIQUE

To gain a basis for estimating a range of times and to enlarge the network predictivity, the planner may use an optional form of time estimation which specifically provides for handling the uncertainties involved in today's accelerated projects. Three time estimates-an optimistic, a most likely, and pessimistic- for each activity could be recorded; these three estimates are the basis for determining the uncertainties involved and the probability that expected events will occur as planned. These duration time estimates can be revised periodically to reflect actual expenditure or changes in the rate of time use [2].

Table 2.1. Comparison of Networking Methods [5].

Methods (1)	Diagraming Method (2)	Network characteristic (3)	Time characteristic (4)	Activity constraints (5)	Major applications (6)	Remarks (7)
CPM	Activities: On branches On nodes (time scale possible on branches)	Additive Time Cost	Fixed or deterministic activity duration	All activities must be completed for project completion	Known projects where duration times can be estimated accurately	Leveling of resources available time/cost tradeoff possible
PERT	Activities: On branches On nodes	Additive Time Cost	Probabilistic activity duration (unimodal Beta function for all activities)	All activities must be completed for project completion	Research and development. Completion times are random variables	PERT/cost available
DCPM	Activities: On nodes (distinction between decision and activity node)	Additive Time Cost	Fixed or deterministic activity duration	Only a set of alternatives must be completed to complete the project	Projects involving decision among many alternatives where duration times are known	Decision node represented by triangle Integer programming solution Heuristic solution
Linear flow graphs	Activities: On nodes	Single multiplicative parameter, (efficiency)	Deterministic (solution possible through use of transforms)	All nodes must be realized to give input/output relations	Projects involving Reliability Feedbacks Repetitions Electrical engineering	Efficient analytical solution and sensitivity analysis through Mason's reduction
GERT	Activities: On branches Conditioned by realization of emanating node	Multiplicative and multiple additive through MGF or transform	Probabilistic (any practical distribution of known MGF)	Realization of network implies realization of set of alternatives only	Stochastic networks Feedbacks Repetitions Uncertainties Research and development Decisions	GERT implements alternative decisions, handles uncertainties, uses flow graphs techniques, synthesizes previous methods

2.6.1 The Optimistic Time Estimate:

This is the estimate of the shortest possible time which an activity can be completed under optimum conditions. The optimistic estimate assumes that the activity is accomplished in the ideal environment, free of even the normal amount of delays or setbacks [2].

2.6.2 The Pessimistic Time Estimate:

This is the estimate of the longest time it might take to complete an activity. The pessimistic estimate assumes that many things go wrong, all of the possible delays or setbacks occur, and everything in general goes badly [2].

2.6.3 The Most-Likely Time Estimate:

This is the estimate which lies between the optimistic and pessimistic; it assumes that normal conditions are encountered in the activity. It anticipates a satisfactory rate of progress but no dramatic breakthrough: in short. "business is as usual [2]."

The differences between the optimistic, pessimistic, and most likely are measured in terms of factors such as people making errors, instructions misunderstanding , and encountering unforeseen technical problems. Estimators exclude certain factors from the estimation process when using the three time estimates. Some of these factors are [2]:

A) Estimates should be based on the actual or potential manpower expected to be available for each activity. They should not include any major manpower changes above the expected level. Estimates should be based upon a uniform workweek, and should be consistent with schedules and budgets if available at this stage. Unless specifically authorized to do so, estimators should not include vacation or other absenteeism, overtime, or extended workweeks.

B) Estimates should be based upon the best present information about the technology, techniques and tools available, as well as about the current scope of the work. Planners should make no allowances for technological breakthrough, dramatic labor, or cost-saving developments, drastic revision of the scope of the job, or improved method expected for standard learning-curve improvements.

C) Managers should not allow for failure on the part of interfacing activities, such as delivery of parts, release of facilities, funding, or work authorizations.

D) The possibility of fire, flood, or local war should not be considered in estimating.

E) It is important not to expand time estimates to fit anticipated budgets as it is to schedule with the network and to avoid expanding or limiting time estimates to fit arbitrary schedules.

When the planner uses three estimates, a single value will be derived to represent the expected activity duration. This is found by using a simple formula [6]:

$$t_e = \frac{a + 4m + b}{6} \quad (2.1)$$

While the variance is found by using the following formula [6]:

$$\sigma_e^2 = \left(\frac{b - a}{6}\right)^2 \quad (2.2)$$

The first formula approximates a mean value for the three estimates. The mean value derived is represented by the symbol t_e (expected activity time); and the optimistic, most-likely, and pessimistic time estimates by a, m, and b, respectively.

2.7 USES OF THE PROBABILISTIC INFORMATION

Once the expected earliest time of an event μ_{Te} and its standard deviation σ_{Te} have been determined, it is possible to use the probability theory to calculate the probabilities of meeting a specific scheduled time for a node. Based on the central limit theorem, the earliest completion for an event is assumed to have a normal probability distribution with a mean of μ_{Te} and a standard deviation of σ_{Te} on the assumption that the path leading to this activity is long enough (i.e. in excess of thirty activities) [6].

For m independent tasks to be performed, $t_1, t_2, \ldots, t_m$ are the actual duration of each of these tasks. These durations are random variables with true means $\mu_1, \mu_2, \ldots, \mu_m$ and true variance $\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2$, and these actual times are unknown until the specified tasks are performed.

For $T = t_1 + t_2 + \ldots + t_m$, T is also a random variable and thus has a distribution. According to the central limit theorem, when m is large and the variables are independent. the distribution of T is approximately normal with mean T, variance V_T, and standard deviation σ_T given by [6]:

$$T = \mu_1 + \mu_2 + \ldots + \mu_m \qquad (2.3)$$

and

$$V_T = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2 \qquad (2.4)$$

$$\sigma_T = \sqrt{V_T} \qquad (2.5)$$

That is the mean of the sum is the sum of the means; the variance of the sum is the sum of the variances; and the distribution of the activity times will be normal regardless of the shape of the distribution of the actual activity performance times [6].

2.8 STUDY OF THE ASSUMPTIONS USED IN THE ANALYSIS AND SOLUTION IN THE PERT NETWORK

PERT has been successfully used in both industry and government for sometime. However, methods and assumptions used in the development of PERT are still subject to constant discussion.

PERT is essentially a network analysis technique. Between each two points in the network are given three time estimates for completion. The two extremes are the optimistic and the pessimistic estimates. The third is the mode of the time distribution. This time distribution is estimated by the β distribution and defined by the function [7]:

$$f(t) = K.(t-a)^{\alpha}.(b-t)^{\beta}, \quad a \leq t \leq b \;\&\; a, b \geq 0 \qquad (2.6)$$

It should be pointed out that the true distribution of the times is unknown, so the above equation is just an approximation. There are, however, some properties of the distribution that are known [7]. The first is that there are two nonnegative abscissa intercepts. The second is that the distribution is continuous. The third is that the distribution must be unimodal. Because beta distribution possesses all of the three qualities, it seems reasonable that it can be used to approximate the true distribution of activity time.

The first group of assumptions that should be considered are those dealing with the specific activity times. The most important of these is the assumption that the beta distribution approximates the true distribution over the range from the optimistic time estimate to the pessimistic time estimate [7].

As was previously pointed out, the beta distribution does have three characteristics of the time estimates, but there are also some inconsistencies . Along with this, it is also necessary to include the corollary assumptions about the mean and the standard deviation [7].

From the equations of the beta distribution, there are four variables (a,b,α,β). The variables a and b are the limits of the distribution. The problem is that there are four unknowns but only three equations to solve. To remedy this the originators of PERT introduced the equation for the standard deviation: (b-a)/6. The question is whether this is the best relationship that can be developed. Assuming that there are two independent random variables that are defined over the interval (a,b). Statistically the standard deviation would be $\sqrt{2}(b-a)$; however, the sum is defined over the region (2a,2b). In this case, the PERT assumption of (b-a)/6 predicts the incorrect result of (2b-2a)/6 [7].

Thus far all the attention has been focused on the activity times in a PERT network, but this is only one part of the problem. There is a significant problem with the analysis of the network as a whole.

The method used to find the project completion time distribution is to find the path that has an expected value not less than that of all the other paths. After this is done, PERT assumes that the project duration is normally distributed with a mean equal to the expected value of the largest path and a standard deviation equal to the standard deviation of the same path. Using these results, the PERT calculated means is most probably lesser than the true mean, and the standard deviation is usually greater than the actual standard deviation [8].

Another possible alternative is to use an entirely different probability distribution such as the gamma distribution which is defined as follows [8]:

$$f(t) = \frac{\delta^{\lambda} t^{\lambda-1} \gamma^{-t\delta}}{\Gamma(\lambda)} \quad 0 < t < \infty \quad \gamma, \lambda > 0 \qquad (2.7)$$

$$= 0 \qquad \text{otherwise}$$

The main advantage of using the gamma distribution is that the parameters can be evaluated by only two time estimates instead of the three that the beta distribution requires. These estimates can be intermediate points which are easier to establish than precise end points of an unknown distribution.

The last alternative deals with this subject of just what time estimates have to be. If the meaning of a and b are changed to some percentage point instead of an absolute extreme, the results should be more accurate. When an extreme is estimated, it is often necessary to extrapolate to get values, but if the 0.025 and 0.975 points of the distribution were to be estimated, it may be easier to get accurate results [8].

2.9 CRASHING IN CPM

Generally, we are faced with the problem of ending the project in a specific period of time; according to the activities normal resources, it will be difficult to end the project according to the specified date, so a time/cost trade off could be used. Utilizing the two time/cost estimates procedure we have the "normal" and "crash" time/cost estimates.

Based upon the relationship between time and money (i.e. the shorter the time the higher the cost, and the longer the time the smaller the cost), these two estimates can be defined as follows [6]:

1. Normal time and cost: The duration to complete the project requiring the least amount of resources or money.

2. Crash time and cost: The minimum possible time to complete the activity and the associated increased cost.

Since each activity is assigned a duration range and related cost, obviously there is a range of durations available for the project depending upon the individual time selections for each activity. Each one of these various project durations also presents a different a project cost.

2.9.1 The Development of Activity Cost Curves:

There is a direct relation between the time and cost for any activity. This relationship takes into account the man force, the resources, method used, and the efficiency achieved. The requirements of the critical path method is not to produce such a curve, but to estimate two important points on the curve. These are the "crash" and "normal" points where [6]:

1. For the "normal" point normal cost is given as the minimum job cost, and the associated minimum time is defined as normal time.

2. For the "crash" point crash time is the minimum possible time, with crash cost being the associated minimum cost.

CPM approximates nonlinear cost curves by a linear function between the normal and the crash durations by assuming a constant slope for the entire activity. This is usually a reasonable approximation of a nonlinear cost curves. If greater precision is desired (and if cost data permits), a more precise approximation of the nonlinear function is possible [6].

2.10 MATHEMATICAL OPTIMIZATION OF THE NETWORK SYSTEM

The basic project representation used in CPM and PERT is the activity arrow diagram. For notational convenience letter i will be associated with each node or event. Then each activity is uniquely defined by a node pair (i,j).

A time/cost curve is assumed to exist for each activity. It will be assumed that all the curves are nondecreasing, convex, and can be approximated by a straight line of the form [6]:

$$Z_{ij} = a_{ij}\, y_{ij} + b_{ij}, \qquad a_{ij} \leq 0,\ b_{ij} \geq 0 \qquad (2.8)$$

Where Z_{ij} is the cost associated with the activity defined by the node pair (i,j), a is the slope and b is the y intercept.

Furthermore, the crash duration d_{ij} and the normal duration D_{ij} obey the relationship:

$$0 \leq d_{ij} \leq y_{ij} \leq D_{ij} \qquad (2.9)$$

the overall project direct cost is found by summing Z_{ij} for all $(i,j) \in$ (project activities) p for the particular set of scheduled y_{ij} of interest thus:

$$\text{Project direct cost} = \sum_{(i,j)\in p}(a_{ij}y_{ij} + b_{ij}) \qquad (2.10)$$

The problem to be considered here may be stated as follows [6]:

$$\text{Minimize} \sum_{(i,j)\in p} Z = \sum_{(i,j)\in p}(a_{ij}y_{ij} + b_{ij}) \qquad (2.11)$$

subject to:

$$0 \le d_{ij} \le y_{ij} \le D_{ij}$$

$$y_{ij} \le t_j - t_i$$

$$t_o = 0,\ t_n = \lambda$$

Where t_i, t_j are the earliest occurrence times for the ith and jth events, and t_n is the earliest occurrence time for the last (nth) event of a project containing n+1 events. The problem is to find a set of y_{ij} and a resulting minimum project direct cost for each possible value of λ in the interval in which λ is defined. The process of generating such schedules is called project expediting [6].

2.11 METHODS USED TO SOLVE MATHEMATICAL FORMULATION

2.11.1 Brute Force:

The initial attempt to expedite a project under the stated

The major difficulty encountered in formulating a parametric linear program is the introduction of the parameter λ. It appears that this can be accomplished if the y_{ij}'s are summed along each path from the initial to the terminal event and restrict the sums to be less than or equal to λ (where λ is the total project duration) [6].

The parametric programming formulation thus becomes [6]:

$$\text{Minimize} \sum_{(i,j)\in p} Z_{i,j} = \sum_{(i,j)\in p} (a_{ij} y_{ij} + b_{ij}) \qquad (2.12)$$

subject to:

$$y_{ij} \le D_{ij} \qquad \text{for all activities}$$

$$y_{ij} \ge d_{ij}$$

$$\sum_{path} ij \le \lambda$$

$$\vdots \qquad \text{for all paths}$$

$$\sum_{path} ij \le \lambda$$

There are standard parametric programming algorithms that can be applied to this formulation. There are, however, some major disadvantages to the parametric programming formulation. It requires that all the paths in p from the initial to the final event be identified. For large projects it is obvious that the total number of paths is very large and it becomes difficult to identify all the paths. A second disadvantage is that, computationally the formulation is not very efficient. The coefficient matrix of the linear programming formulation consists mostly of zeros so that much of the computation deals with zero elements which contain no useful information concerning the project [6].

2.11.3 The Flow Approach:

After formulating the primal linear program, a dual program having the characteristics of the flow program can be derived. The flow program is then solved by efficient flow algorithms rather than by the highly inefficient standard linear programming methods.

In 2.11.2 a linear programming formulation of CPM problem was given. A slightly different formulation in this part will be used. First the objective function will be considered. The cost of each activity will be described as follows [6]:

$$\text{Activity } (i,j) \text{ direct cost} = K_{ij} - C_{ij}x_{ij}$$

where:

K_{ij} is the y intercept of the cost curve for activity (i,j),

C_{ij} is the absolute value of the slope of the cost curve for activity (i,j), and

x_{ij} is the duration of activity (i,j).

The objective is to minimize the sum of all the activity costs which is expressed as follows:

$$\text{Total direct cost} = \sum_i \sum_j (K_{ij} - C_{ij}\, x_{ij})$$

Since the sum of the K_{ij} terms is constant, the minimization of the total direct project costs can be accomplished if we maximize the following expression:

$$\sum_i \sum_j C_{ij} x_{ij}$$

The linear programming formulation becomes [6]:

$$\text{Maximize } f(x) = \sum_i \sum_j C_{ij} x_{ij} \qquad (2.13a)$$

subject to:

$$T_i + x_{ij} - T_j \le 0 \qquad \text{for all } ij \qquad (2.13b)$$

$$x_{ij} \le D_{ij} \qquad \text{for all } ij \qquad (2.13c)$$

$$-x_{ij} \le -d_{ij} \qquad \text{for all } ij \qquad (2.13d)$$

$$T_n - T_1 \le \tau \qquad (2.13e)$$

Where:

T_k denotes the earliest expected time for node k to be realized,

D_{ij} is the normal duration for activity (i,j),

d_{ij} is the crash duration for activity (i,j), and

τ is a parametric variable which is used to restrict the project duration.

The first set of constraints (2.12b) apply to each activity (i,j) and state the difference between the earliest node times, T_i and T_j, must be at least x_{ij}, which is the scheduled duration of activity (i,j). Equations (2.12c) and (2.12d) imply that x_{ij} is constrained to lay between its normal and crash time. Finally Equation (2.12e) guarantees that the time between the realization of the start node, T_1, and the realization of the sink node, T_n, is less than or equal to the project duration τ [6].

As discussed previously, this problem could be solved by the simplex method, which finds the optimum schedule for different values of τ. The simplex procedure will prove less efficient than the flow algorithm that will now be developed.

The first step in the development of the flow algorithm to solve this system is to find the dual of the problem just presented. Different dual variables will be used for each set of equations in the primal problem. f_{ij}, v_{ij}, and w_{ij} correspond to Equations (2.13b), (2.13c), and (2.13d), respectively, while the dual variable for Equation (2.13e) is y. Since all the primal variables are unrestricted in sign and the primal constraints are all inequalities, the dual constraints are equalities and the dual variables are all sign-constrained. The dual formulation becomes [6]:

Minimize $g(f,v,w,y) = \tau y + \sum_i \sum_j D_{ij} v_{ij} - \sum_i \sum_j d_{ij} w_{ij}$ (2.14a)

subject to:

$$\sum_j (f_{ij} - f_{ij}) = \begin{bmatrix} y; & i = 1 \\ 0; & i = 1,n \\ -y; & i = n \end{bmatrix} \qquad (2.14b)$$

$$f_{ij} + v_{ij} - w_{ij} = c_{ij} \qquad \text{for all } ij \qquad (2.14c)$$

$$f_{ij}, v_{ij}, w_{ij}, y \geq 0 \qquad \text{for all } ij$$

The first set of constraint Eq. (2.13b) has a flow interpretation. The equation imply that there is a flow of y out of the source node and an equal flow into the sink. All intermediate nodes have conservation of flow.

Recently, a new approach has been suggested to solve the maximum flow problem that does not construct layered networks but instead maintains distance labels [9]. In formally a distance label of a node is an integral lower bound on the length of the shortest augmenting path from that node to the sink. A distance label is called exact if it equals the length of the shortest augmenting path. If a distance label is not guaranteed to be exact, then it is called approximate. The algorithms that utilize distance labels are referred to as distance-directed algorithms [9].

2.12 OPTIMAL TIME-COST TRADE-OFF IN GERT NETWORK

The problem of optimal-cost trade-off in a deterministic networks can be simply stated as of optimally investing a fixed amount of money to achieve the maximal reduction in the duration of the project. Alternatively, the problem may consist of calculating the minimum additional investment needed to reduce the duration of the project (or a subset

2.13 RESEARCH METHODOLOGY

For achieving the objective of this thesis, the following research plan is to be followed in order to investigate the economics of reducing the variance and/or the expected value of the duration of the project to achieve better prediction:

1- Identify and formulate the problem mentioned in writing.

2- Identify the constant parameters and variables involved through introducing symbols to represent each one.

3- Select the variables that appear to be most influential.

4- State the relationships among the variables based upon known principles.

5- Construct the model by combining all relationships into a system of symbolic relationships.

6- Perform symbolic manipulations.

7- Derive solutions from the model.

8- Test the model.

9- Revise the model.

If the resultant model was difficult to be solved, a heuristic would be developed to speed up the process of reaching the optimum solution or to find a good solution not guaranteed to be optimal.

2.14 SUMMARY

Due to the scarcity, or even the absence of the subjects dealing with the crashing in PERT networks, The subjects discussed in the chapter, with the exception of El-Maghraby attempt, are lacking previous work on this subject. The topics are merely basic concepts on CPM and PERT.

At the beginning a brief historical review of CPM, PERT, and SANs are introduced followed by comparisons between networking techniques. Concerning PERT networks, several topics are presented, these topics involve: Identification of the three time estimate technique, the uses of the probabilistic information, and the assumptions used in the analysis and solution in the PERT networks. The concept of crashing with the development of activity cost curves are then introduced. Three methods: The Brute Force, the Linear Programming Approach, and the Flow Approach, are used as an example for the methods used to solve mathematical formulation. As an example of stochastic networks, the optimal time-cost trade-off in GERT networks is presented.

The last section identify the research methodology that will be followed for achieving the objective of the thesis.

CHAPTER THREE

EVALUATION OF PROJECTS UNDER RISK AND UNCERTAINTY CONDITIONS

Since the model that will be developed is of probabilistic nature, it is worthwhile to bring up the subject of risk and uncertainty.

3.1 DIFFERENCE BETWEEN RISK AND UNCERTAINTY

The classical distinction between risk and uncertainty is that an element or analysis involves risk if the probabilities of the alternative, possible outcomes are known, while it is characterized by uncertainty if the frequency distribution of the possible outcomes is not known [11]. The distinction between conditions of assumed certainty, risk, and uncertainty for a project life is portrayed graphically in Figure 3.1.

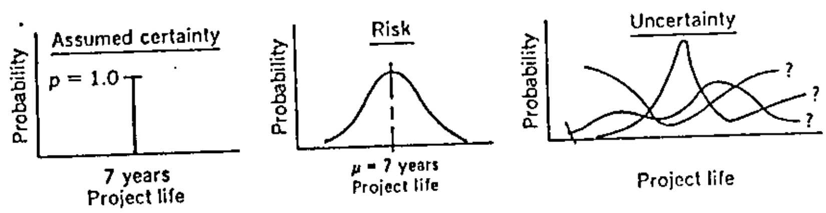

Figure 3.1. Illustrations of Assumed Certainty, Risk, and Uncertainty as Applied to Life of a Project [11].

Another less restrictive distinction between risk and uncertainty is that risk is the dispersion of the probability

3.3 RISK AND RETURN TRADE OFF

It is generally accepted that the riskier a project, the higher the apparent return it must promise to warrant acceptance. It would be desirable to determine differential risk allowances which would reduce all project to common basis. This cannot be done precisely, however, for the statement of differential risk allowances is very much a matter of subjective judgment [11].

Before a firm can make investment decision to include allowances for risk, the firm's policy toward risk should be determined. The amount of a risk a firm is prepared to undertake to secure a given actual or apparent monetary return is a general question of values. There is no rational or logical criterion by which the choice can be made. Rather, this is largely a function of the preferences of the decision-maker of the firm and the amount of risk to which the firm is already exposed [11].

In general, the relationship between the expected return, which is a function of the expected duration, and risk (degree of variability of the return) can be represented as in Figure 3.2.

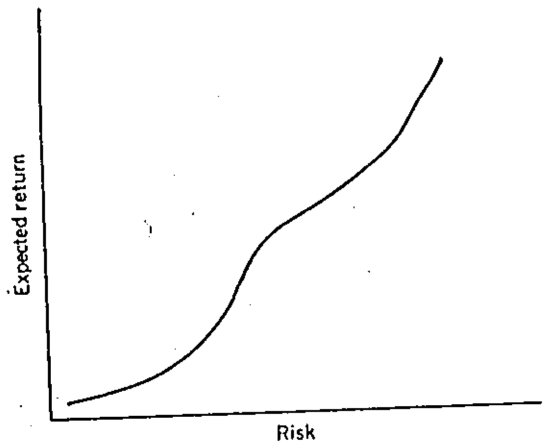

Figure 3.2. Relationship Between Return and Risk.[11]

3.4 EFFECTS OF RISK AND UNCERTAINTIES ON CAPITAL PROJECT

There are a number of procedures for describing analytically the effect of risk and uncertainty on capital projects. Such procedures are generally categorized as sensitivity or risk analysis.

Sensitivity analyses are performed when conditions of uncertainty exist for one or more parameters. The objectives of a sensitivity analysis are to provide the decision-maker with information concerning the behavior of the measure of economic effectiveness due to errors in estimating various values of parameters and the potential for reversals in the preferences for economic investment alternatives [12].

Typical parameters for which conditions of risk can reasonably be expected to exist include the initial investment, yearly operating and maintenance expenses, salvage values, the life of an investment, the planning horizon, and the minimum attractive rate of return. The parameters can be statistically independent, correlated with time and/or correlated with each other [12].

3.5 DECISION CRITERIA AND METHOD FOR RISK ANALYSIS.

Several decision criteria will be described, in which most of the criteria apply to classical risk problems in which probabilities of various outcome can be estimated, however, the first two criteria can be applied to decision problems even when the probabilities are unknown [11].

A) Dominance criterion or elimination check. The first step in making a decision when the results for all alternatives and states of nature can be quantified is to eliminate from consideration any unpreferred alternatives irrespective to its state of nature. If the result for any alternative, X, is better than the result than some other alternative, Y, for all possible state of nature, then alternative X is said to dominate alternative Y, and thus Y can be dropped from further consideration.

B) Aspiration level criterion. The aspiration level criterion involves selecting some level of aspiration and then choosing so as to maximize or minimize the probability of achieving this level. An aspiration level is simply some level of achievement (like profit) the decision-maker desires to attain or some level of negative results (like cost) to be avoided.

C) The most probable future. The most probable future criterion suggests that as the decision-maker considers the various possible outcomes in a decision, he overlooks all except the most probable one and act as though it is certain. Many decisions are based on this principle, since, in fact, only the most probable future is seriously considered.

D) Expected value criterion. Using the expectation principle and thereby choosing so as to optimize the expected payoff or cost simplifies a decision situation by weight in dollar payoffs or costs by their probabilities. The criterion is often known as the expected monetary value. As long as the dollar consequences of possible outcomes for each alternative are not very large in the eyes of the decision-maker, the expectation principle can be expected to be consistent with a decision-maker behavior.

The general formula for finding the expected outcome of a variable x for any alternative, A_i, having k discrete outcomes is [11]:

$$E[x] = \sum_{j=1}^{k} x_j . p(x_j) \qquad (3.1)$$

where

E[x] = Expected value of x

x_j = jth outcome of x

$p(x_j)$ = Probability x_j occurring.

E) Expected-variance criterion. The expected-variance criterion or procedure, involves reducing the economic desirability of a project into a single measure which indicates consideration of the expected outcome as well as variation of that outcome.

One single example is [11]:

$$Q = E[x] - A.\sigma[x] \qquad (3.2)$$

Where

Q = Expectation-variance measure

E[x] = Mean or expected monetary outcome

σ[x] = Standard deviation of monetary outcome

A = Coefficient of risk aversion

The variance of a variable, x, for any alternative having k discrete outcomes is [11]:

$$V[x] = \sum_{j=1}^{k} (x_j - E[x])^2 \, p(x_j) \qquad (3.3)$$

Where V[x] = variance of x and all the other symbols were defined with Eq. (3.1).

There are innumerable other expectation-variance criteria which can be applied depending on the risk preferences and sophistication of the decision-maker.

F) Certain monetary equivalence criterion. An offshoot of the expectation-variance criterion is to be subjectively determined the certain monetary equivalence of any set of results for any alternatives. The certain monetary equivalence is merely the monetary amount for certain at which the decision-maker would be indifferent between that amount and various possible monetary outcomes. This concept is very useful in practice and can be applied to situations involving gains or losses. while it can be most meaningfully applied to risk situations in which various payoffs or cost and their respective probabilities are known, it can also be used in situations involving uncertainty regarding payoffs/costs, or probabilities, or both.

G) Expected utility criterion. The expected utility criterion or method has particular usefulness for analyzing projects in which the potential gain or loss of significant size compared to the total funds available to the firm. Most specifically, if the marginal utility or desirability of each dollar potentially to be gained or lost is not a constant, then the utility of dollars is relevant, and then it may be worthwhile to use the expected utility method rather than the probabilistic monetary method.

The expected utility method consists of determining the cardinal utility, such as the relative degree of usefulness or desirability to the decision-maker, of each of the possible outcomes of a project or group of projects on some numerical scale and then calculating the expected value of the utility to use as the measure of merit.

3.6 EXPECTED VALUE OF SAMPLE INFORMATION

3.6.1 General Model for Risk and Uncertainty Problems:

In order to provide a framework for the subsequent discussion, it is useful to employ a general model of decision problems in which there are various possible outcomes (called states of nature) in combination with several alternative actions. As depicted in Table 4.1, m mutually exclusive investments alternatives, $\{A_t\}$, and k mutually exclusive and collectively states of nature, $\{S_j\}$, have been identified. The outcome, R_{ij}, is normally expressed in equivalent durations, return, or costs, but it can be in any measure. This tabular model is often called a payoff table [11].

Referring to the previous table, for every alternatives there are different state of nature and there respective probabilities. These probabilities were put with no previous information regarding the future. Giving additional study or sample information the respective probabilities will be changed in a matter related to the additional information, resulting in the expected value of sample information (EVSI).

Table 3.1. General Model for Risk and Uncertainties Problems

Alternatives	State of nature (Probability of state) S_1 $P(S_1)$	S_2... $P(S_2)$	S_j... $P(S_j)$	S_k $P(S_k)$
A_1	R_{11}	R_{12}	R_{1j}	R_{1k}
A_2	R_{21}	R_{22}	R_{2j}	R_{2k}
⋮	⋮	⋮	⋮	⋮
A_t	R_{t1}	R_{t2}	R_{tj}	R_{tk}
⋮	⋮	⋮	⋮	⋮
A_m	R_{m1}	R_{m2}	R_{mj}	R_{mk}

The maximum possible EVSI is the expected value of perfect information (EVPI) and the expected loss due to imperfect information as to what will be the state of nature in a situation involving risk. Interpreted another way, the expected value of perfect information is the amount which would be gained, on the average, if the future regarding a particular situation became perfectly predictable and decision changed to the optimal choices based on the new known conditions [11].

3.7 RISK INTERPRETATION FOR PERT NETWORKS

As stated previously, risk is the dispersion of the probability distribution of the element being estimated, and since in PERT the durations are random variables having expected value and variance, the variance is considered as the risk associated with that activities.

The optimistic, pessimistic, and the most likely estimates can be depicted in a way similar to Table 3.1 In other word, the three estimates could be viewed as three states of nature or three possible outcomes in combination with several alternative actions. As depicted in Table 3.2 n mutually exclusive durations alternatives, $\{B_i\}$, and three mutually exclusive and collectively states of nature. The combination of action B_i and the state of nature yields a net result D_{ij}, where D_{ij} is expressed in time unites.

Table 3.2. General Model for Risk in PERT Activities

	State of nature (Probabilty of state)		
Alternatives	Optimistic 1/6	Most likely 4/6	Pessimistic 1/6
B1	D11	D12	D13
B2	D21	D22	D23
⋮	⋮	⋮	⋮
Bt	Dt1	Dt2	Dt3
⋮	⋮	⋮	⋮
Bn	Dn1	Dn2	Dn3

where the expected value for each alternative is

$$E[B_t] = 1/6(D_{t1}) + 4/6(D_{t2}) + 1/6(D_{t3}) \qquad (3.4)$$

Which is exactly the same as the ordinary expected value of PERT activities.

Adding more information or future regarding a particular situation becomes predictable, changes the previous probabilities in such a way that the optimistic and pessimistic duration will be more close to each other.

Since the variance depends on the difference between the optimistic and the pessimistic duration estimates, the variance will be reduced as a result of adding additional information and tantamount the risk will be reduced.

The effect of reducing the variance by additional information and other factors on the cost, will be cleared in Chapter Four.

3.8 SUMMARY

Project initialization needs activities identifications. For each activity, several alternatives might be available. The choice of the best alternative needs evaluation of alternatives under risk and uncertainties conditions.

Risk is the dispersions of the probability distribution function of the element being estimated or calculated outcomes being considered.

The chapter includes the difference between risk and uncertainties, causes of risk and uncertainties, risk and uncertainties trade off, the effects of risk and uncertainties on capital projects, and the expected value of sample information. A special section is devoted for the sake of introducing the different decision criteria and methods used for risk analysis. The last section illustrates how the risk can be interpreted for PERT networks and how the expected value of the beta distribution can be related to the expected utility criterion.

CHAPTER FOUR

PROBLEM FORMULATION

Recalling our objective, which is the determination of the optimal allocation of a fixed amount of funds K among the remaining activities such that the probability of realizing the terminal node on or before a specified time is maximized.

The same objective can be stated differently as achieving a predetermined target probability of realizing the terminal node on or before a specified time with the minimum amount of fund K allocated among the remaining activities.

4.1 GENERAL ASSUMPTIONS

Referring to Eqs (2.3) and (2.4) and the assumptions involved, the probability of meeting a schedule time T_s for a particular event can be calculated by visualizing a normal distribution centered at μ_{TE} as illustrated in Figure 5.1. The probability of meeting the desired scheduled time T_s is obtained by finding the area under the normal curve to the left of T_s.

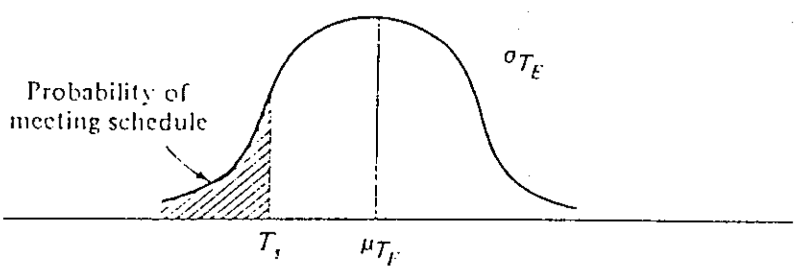

Figure 4.1. Calculating the Probability of Meeting Schedule [6].

The familiar Z statistic of the normal distribution can be calculated by realizing that $T_s = \mu_{TE} + Z\sigma_{TE}$. Therefore [6]:

$$Z = \frac{T_s - \mu_{TE}}{\sigma_{TE}} \qquad (4.1)$$

The Z value can be converted to a probability by means of Table A.1.

Considering the previous equation, and fixing the value of T_s, the Z value is function of both the expected value and the variance. Increasing the probability necessitates the reduction of the expected value as well as the variance (the risk involved).

Since the variance and the expected value are independent, each one will be investigated alone, followed by an integrating model consisting of the two.

4.1.1 Expected Value:

The assumptions that will be used in this section, are the same assumptions used by El-Maghraby [10] GERT model mentioned in section 2.12.

In a manner reminiscent of the study of optimal time cost trade-off in the Deterministic Activity Network, it will be assumed that each activity (ij) in the network has an associated "normal" expected duration, μ_{ij}, and a "crash" expected duration, l_{ij}. The words "expected duration" are used advisedly to indicate the mathematical expectation of a random variable (RV). That is to say, under "normal" pace of operation the actual duration of the activity is a RV whose expected value is given by μ_{ij}. On the other hand, under "crash" pace of operation, the actual duration of the activity is also a RV variable whose expected value is l_{ij} [10].

It is further assumed that the cost of performing the job varies with (i.e., is a function of) the actual duration of the activity. Specifically, the cost C_{ij}, is a monotone nondecreasing function of the duration Y_{ij} [10];

$$C_{ij} = \psi \; (Y_{ij}) \qquad (4.2)$$

Since Y_{ij} and l_{ij} are assumed to be RVs, so will be C_{ij}. Denote its expected value by $E\,[C_{ij}] = \gamma_{ij}$.

Now suppose that an additional amount r_{ij} is invested in activity (ij) in order to shift its level from one (slow) level to a faster one. Then the distribution of the activity duration is expected to be shifted to the left by some amount depending on the sensitivity of the average duration to investment. It is worthwhile to say that if such shift in average duration is not realized for any $r_{ij} > 0$, it would be futile to invest any sum in the first place. Let the decrease in the average duration be a function ϕ of the additional investment r_{ij}; hence the new mean duration is given by[10]:

$$\hat{\mu}_{ij} = E\,[\hat{Y}_{ij}] = \mu_{ij} - \phi(r_{ij}) \qquad (4.3)$$

While the new expected cost denoted by $E\,[\hat{C}_{ij}(r_{ij})]$, is [10]:

$$\hat{\gamma}_{ij} = E\,[\hat{C}_{ij}(r_{ij})] = E\,[\psi_{ij}(\hat{Y}_{ij})] + r_{ij} \qquad (4.4)$$

Equation (4.4) is different from the Equation (2.8) concerning the Deterministic Activity Network, in that the cost in the former equation is function of two variables; the additional investment r_{ij} which results in shifting the expected duration to a faster level and the actual cost of this activity at this level which is still a function given by Equation (4.2). The relation among the various parameters are depicted in Figure 4.2.

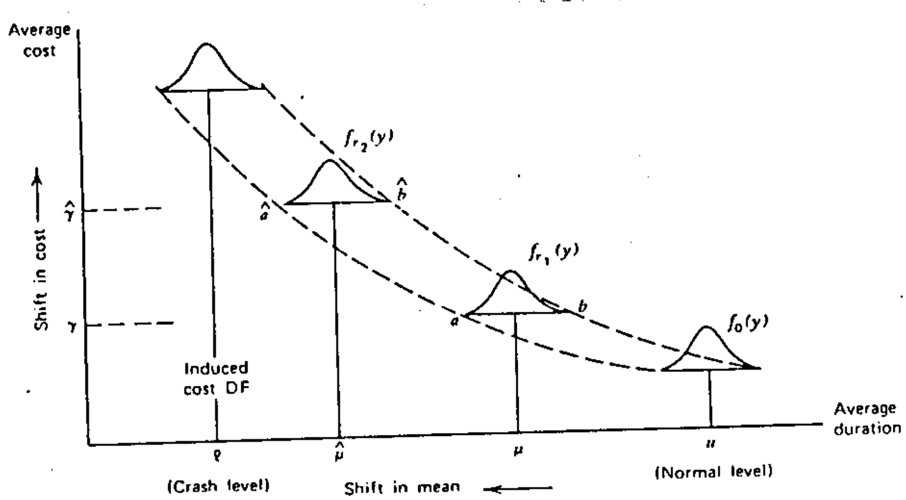

Figure 4.2. Relations Between μ_{ij} and γ_{ij} [10].

Developing more precisely the concept of the additional investment r_{ij} in activity (ij).

Suppose that the duration Y possesses a Probability Density Function (PDF) $F_r(y)$, where the subscript r emphasizes the dependence of F on the level of the additional investment r where $F_o(y)$ is the raw density function. Hence the expected cost is given by [10]:

$$\gamma = \int_a^b \psi\ (y)\ dF_o(y) \tag{4.5}$$

Where a and b are the limits on the values of the duration y.

If an investment r is made in that activity, the new average cost is given by [10]:

$$\hat{\gamma} = r + \int_{\hat{a}}^{\hat{b}} \psi\ (\hat{y})\ d\hat{F}_r(\hat{y}) \tag{4.6}$$

Which may be larger or smaller than γ.

Considering the case in which $\hat{\gamma} < \gamma$. This means that even after an activity (ij) has received an additional investment r, its expected cost is less than its cost before the additional investment. Then it must be true that the savings in cost due to the reduction in time more than offset the additional investment of r. Therefore there must exist an initial investment r′ corresponding to the minimal cost

γ°[10],

$$\gamma^{\circ} = \min_{r} [\ r + \int_{\hat{a}}^{\hat{b}} \psi\ (\hat{y})\ d\hat{F}_{r}(\hat{y})\ ]; \qquad 0 < r < \infty \qquad (4.7)$$

So activity (ij) should be invested in by the additional amount r′ immediately regardless of any desired reduction in the expected duration of the whole project.

As a result of the above analysis, it is assumed that the "zero" level of any additional investment incorporated r′, such that the average cost of the activity is always increased. Furthermore, an additional investment in one or more activities is made for the explicit purpose of reducing the total duration to the realization of a specific terminal node.

To avoid the above mentioned discussion, It is assumed that the the functions ϕ and ψ can be approximated in the region of interest by linear functions with negative slopes [10].

Following the previous discussion, the means $\hat{\mu}$ of Eq. (4.3) becomes [10]:

$$\hat{\mu} = \mu + qr; \qquad -\mu/r < q < 0 \qquad (4.8)$$

and $\hat{\gamma}$ of Eq. (5.4) can be derived as follows [10]:

$$\hat{\gamma} = E\ [m_o + m\hat{Y}_{1j}] + r \tag{4.9a}$$

$$\hat{\gamma} = E\ [m_o] + mE\ [\hat{Y}_{1j}] + r \tag{4.9b}$$

substitute the value of $E\ [\hat{Y}_{1j}] = \hat{\mu} = \mu + qr$ in (4.8-b)

$$\hat{\gamma} = m_o + m\ (\mu + qr) + r \tag{4.9c}$$

Which is equivalent to:

$$\hat{\gamma} = m_o + m(\hat{\mu}) + r \tag{4.9d}$$

Eq. (4.8c) can be rewrite as follows:

$$\hat{\gamma} = m_o + m\mu + mqr + r \tag{4.9e}$$

or

$$\hat{\gamma} = \gamma + r(mq + 1)$$

$$\hat{\gamma} = \gamma + br; \quad b = mq + 1 \tag{4.9f}$$

Where $q < 0$ is the marginal decrease in the level of the activity per unit increase in investment in the activity, m, and m_0 are the linear slope and the y intercept of the function ψ respectively.

For the linear case as given by Eqs (4.8) and (4.9f), it is possible to develop explicit expressions for the expected duration to the realization of node n_l $E[\hat{T}_{nl}]$, and the expected cost of reaching n_l $E[\hat{C}_{ij}]$ in terms of the investments $\{r_{ij}\}$ in activities (ij).

Let π^k denote the kth path from starting node 1 to terminal node n_l, and Y^k denote the duration of path π^k.Clearly, taking into consideration the central limit theorem, Y^k is the sum of the independent RVs $\{Y_{ij}\}$ of the duration of activities along π^k [10],

$$Y^k = \sum_{(ij)\in \pi^k} Y_{ij}$$

$$E[T_{nl}]^k = E[\Sigma Y_{ij}]$$

$$= \Sigma(E[Y_{ij}])$$

$$= \sum_{(ij)\in \pi^k} \mu_{ij} \qquad (4.10)$$

Now replacing μ_{ij} with $\hat{\mu}_{ij} = \mu_{ij} + q_{ij}r_{ij}$, and with simplification the following is obtained [10]:

$$\hat{\mu}_{nl} = E[\hat{T}_{nl}] = E[T_{nl}] + \sum_{(ij)\in \pi^k} q_{ij} r_{ij}$$

$$\hat{\mu}_{nl} = \sum_{(ij)\in \pi^k} \mu_{ij} + \sum_{(ij)\in \pi^k} q_{ij}r_{ij} \qquad (4.11)$$

Similarly [10],

$$\hat{\gamma} = \sum_{(ij)\in \pi_k} \gamma_{ij} + \sum_{(ij)\in \pi_k} b_{ij} r_{ij} \qquad (4.12)$$

Where Eqs (4.11) and (4.12) are the maximum expected duration over all the paths and the expected cost associated with this path respectively.

The initial expected value is calculated according to the following Eqs.

$$\mu_{ij} = E\,[Y_{ij}] = \Sigma\, Y_{ij} p(Y_{ij}) \qquad (4.13)$$

for discrete RVs, and

$$\mu_{ij} = E\,[Y_{ij}] = \int_a^b Y_{ij}\ F(Y_{ij}) dY_{ij} \qquad (4.14)$$

for continuous random variables.

4.1.2 Variance:

As was mentioned above the probability is indirectly proportional with the value of the variance or the risk involved. The variance can be eliminated when the state of nature is predictable and this, of course, will need additional information, and its financial consequences or (cost of information). The reduction in variance is proportional to the amount of cost that is function of the additional information.

The additional information could come from a new precise instrument for better prediction of weather conditions, expert people in the field of activity, "Spy" satellites, and remote sensing equipment.
Obviously, these information have an initial investment and operating cost.

The variance equations are derived in a similar manner as the expected value, where the additional amount r_{ij} is invested in activity (ij) in order to reduce the variance. The variance will be reduced by some amount depending on the sensitivity of the variance on the investment. Letting the decrease in the variance be a function α of the additional investment r_{ij}, hence, the new level of risk is given by:

$$\hat{\sigma}^2 = \sigma^2 - \alpha(r_{ij}) \tag{4.15}$$

If the function α can be approximated in the region of interest by a linear function, then the new variance of Eq. (4.15) becomes:

$$\hat{\sigma}^2 = \sigma^2 + sr \quad ; \qquad -\sigma^2/r < s < 0 \tag{4.16}$$

Where $s < 0$ is the marginal decrease in the level of the variance per unit increase in the investment.

In a similar manner the new cost associated with reduction of the variance is:

$$\hat{\beta} = n_o + n\sigma^2_{ij} + nsr + r \qquad (4.17)$$

where n and n_o are the linear slope and the y intercept of a linear function δ, in the region of interests, which relates the cost with the variance.

The total amount invested for each activity will equal the amount invested in order to reduce the duration plus the amount invested in the reduction of the variance. The sum of the two investment should be less than or equal to the amount available for that activity.

4.2 CONSTRUCTING THE MODEL

The three basic steps in a constructing a linear programming model are as follows:

Step I: Identify the unknown variables to be determined (decision variables), and represent them in terms of algebraic symbols.

Step II: Identify all the restrictions or constraints in the problem and express them as linear equations, if the formulation is linear, or inequalities which are functions of the unknown variables.

Step III: Identify the objective or criterion and represent it as a function of the decision variables, which is to be maximized or minimized.

Three different formulation alternatives are introduced. Each will be discussed in terms of:

A) Application.

C) Advantages and shortcomings.

D) Time and ease of solution.

4.2.1 First Alternative:

In this alternative, the Flow-Network approach is adopted. Activity Networks of concern possess no "flow" of a uniform commodity through them. In fact the network is a representation of heterogeneous number of activity or tasks that combine to realize certain events. However for the purpose of determining the critical path, it is possible to view the network as a flow network in which a unit flow enters the origin 1 and exits at the terminal n. The duration of each activity Y_{ij}, can be interpreted as the time of transportation of the commodity from node i to node j or, better still, as the utility of such transportation. Naturally, all intermediate nodes act transshipment centers. Under such interpretation, determining the critical path, which is the longest path, is equivalent to determining the path of maximum utility from 1 to n [10].

Step I: Identify the Decision Variables. The unknown activities to be determined are the amount invested on each activity for expected value, and variance reduction. Representing them by algebraic symbols.

re_{ij}——Amount invested for expected value reduction in activity (ij).

rv_{ij}——Amount invested for variance reduction in activity (ij).

Step II: In this problem the first constraint is the limited availability of the total amount that should be invested.

$$\sum_{(ij)} (re_{ij} + rv_{ij}) = K \tag{4.18}$$

where K is a fixed amount of fund available for the project.

Similarly, there is an upper limit on the amount to be invested in activity (ij).

$$0 \leq re_{ij} \leq \overline{re}_{ij} \tag{4.19}$$

$$0 \leq rv_{ij} \leq \overline{rv}_{ij} \tag{4.20}$$

where $\overline{re}_{ij}$, and $\overline{rv}_{ij}$ are the upper limits on the amount to be invested in activity (ij) for expected value and variance reduction respectively.

In addition, there are the flow conservative constraints, in which the convention that flow into a node is negative and flow out of a node is positive is adopted.

$$x_{ij} \geq 0 \qquad (4.21a)$$

$$\sum x_{1i} = 1 \qquad (4.21b)$$

for all set of nodes i connecting from 1 (i.e., occur after)

$$-\sum x_{ij} + \sum x_{jk} = 0, \qquad j = 2,3,...,n-1 \qquad (4.21c)$$

for all set of nodes i connecting to j and k (i.e. occur immediately before j and k)

$$-\sum x_{in} = -1 \qquad (4.21d)$$

for all set of nodes i connecting to n (i.e. occur immediately before n)

Step III: The objective function at this stage is to minimize the probability, in order to find the most critical path (i.e. least probable) subject to the above mentioned constraints.

The above formulation will give, at the end, the path with the minimum Z value (i.e. minimum probability), when we invest the total amount available K.

After the identification of this critical path, then this path should be maximized in term of the probability and the total fund. So the formulation becomes as follows:

$$\text{Maximize} \quad Z \quad = \quad \frac{T - \sum_{(ij)\in c} (\hat{\mu}_{ij})}{\sqrt{\sum (\hat{\sigma}^2_{ij})}}$$

Subject to

$$\sum re_{ij} + \sum rv_{ij} \leq K$$

$$0 \leq re_{ij} \leq \overline{re}_{ij}$$

$$0 \leq rv_{ij} \leq \overline{rv}_{ij}$$

where c is the critical path found from the initial formulation.

A) Application. This formulation is best suitable for the cases where there is one critical path, in other words, when all the paths have probability greater than the required probability except one path. So the most critical path is first identified by the first formulation followed by the next formulation where the same critical path is maximized.

B) Advantages and shortcomings. The advantages of such an alternative are two: First, the whole body of literature on network flows and LP, which is important, can be utilized to answer many questions (e.g., sensitivity analysis to activity durations and variance) deemed of interest in the study of such networks; Second,other advantage that can be viewed here, is that the formulation does not include any equation, either in the objective function or in the constraints, that is needed to specify all the paths.

The shortcomings can be viewed from its place of application. This model will not work if the network includes more than one critical path.

C) Time and ease of solution. Since the problem is not linear, then some difficulties will be faced in trying to solve it. But the relative time of solution will smaller than the other forthcoming formulation, since identification of every path is not needed.

4.2.2 Second Alternative:

In this formulation, integration of the two previous formulations will be presented.

Step I: The decision variables are exactly the same as the previous decision variables.

Step II: In this alternative the first constraint is also the limited availability of the total amount invested.

$$\sum_{(ij)} re_{ij} + \sum_{(ij)} rv_{ij} \leq K \qquad (4.23)$$

Similarly, there is an upper limit for the amount to be invested in activity (ij).

$$0 \leq re_{ij} \leq \overline{re}_{ij}$$

$$0 \leq rv_{ij} \leq \overline{rv}_{ij}$$

Step III: The objective function is to maximize the path with the minimum probability.

Maximize Z = minimum of

$$\left[\frac{\left(T - \sum_{(ij)\in p1} \hat{\mu}_{ij}\right)_{p1}}{\sqrt{\sum_{(ij)\in p1} \hat{\sigma}^2_{ij}}},\ \frac{\left(T - \sum_{(ij)\in p2} \hat{\mu}_{ij}\right)_{p2}}{\sqrt{\sum_{(ij)\in p2} \hat{\sigma}^2_{ij}}},\ \ldots\ldots,\ \frac{\left(T - \sum_{(ij)\in pn} \hat{\mu}_{ij}\right)_{pn}}{\sqrt{\sum_{(ij)\in pn} \hat{\sigma}^2_{ij}}} \right]$$

where p1, p2,...,pn are all the routes from the initial node to the terminal node.

The formulation becomes as follows:

Maximize Z = minimum of

$$\left[\frac{\left(T - \sum_{(ij)\in p1} \hat{\mu}_{ij} \right)_{p1}}{\sqrt{\sum_{(ij)\in p1} \hat{\sigma}^2_{ij}}}, \frac{\left(T - \sum_{(ij)\in p2} \hat{\mu}_{ij} \right)_{p2}}{\sqrt{\sum_{(ij)\in p2} \hat{\sigma}^2_{ij}}}, \ldots\ldots, \frac{\left(T - \sum_{(ij)\in pn} \hat{\mu}_{ij} \right)_{pn}}{\sqrt{\sum_{(ij)\in pn} \hat{\sigma}^2_{ij}}} \right]$$

Subject to

$$\sum_{(ij)} re_{ij} + \sum_{(ij)} rv_{ij} \leq K$$

$$0 \leq re_{ij} \leq \overline{re}_{ij}$$

$$0 \leq rv_{ij} \leq \overline{rv}_{ij}$$

A) Application. This formulation is treated according to goal programming, which is a very useful tool for dealing with problems where several objectives must be considered simultaneously. However, it does not require establishing goals for all of the objectives, and it is not always possible to do this in a meaningful way. In particular, some objectives are open-ended and it is required to make as much progress toward them as possible. In this case our several objectives are to maximize the probability for each path as much as possible and this will be achieved by maximizing the minimum progress toward all objectives. This formulation is an exact one, and if solved will give us the required information.

B) Advantages and shortcomings. As mentioned above the advantage of this formulation is that it gives us an exact solution for the problem.

The shortcomings come: First, from the need of identification of each route in the objective function; second, the nonlinearity in the objective function.

C) Time and ease of solution. Concerning this alternative, the time factor will depend on the number of paths, in other words, the complexity of the network that will affect the time taken to solve the problem.

The difficulties in solving this alternative arise from the nonlinearity in the objective functions which comes from two factors: First, the overall objective function in its present state

$$\text{Maximize} \quad Z = \text{minimum } \{Z_1, Z_2, \ldots, Z_k\}$$

certainly does not fit into a linear programming format. This problem can be reformulated as follows [13]:

1. An artificial variable z is introduced, to represent the minimum value among the K objectives.

$$z = \text{minimum } \{Z_1, Z_2, \ldots, Z_k\}.$$

By introducing the artificial variable the overall objective function can be written as

$$\text{Maximize } Z = z,$$

which is a legitimate linear programming objective function. The remaining question is how to incorporate the definition of z directly into a linear programming model. The definition implies that[13]:

$$z \le \sum_{j=1}^{n} c_{j1} x_j$$

$$z \le \sum_{j=1}^{n} c_{j2} x_j$$

$$\vdots$$

$$z \le \sum_{j=1}^{n} c_{jk} x_j,$$

each of the above constraints represent the probability of its respective route, in terms of the Z statistics.

These inequalities are legitimate linear programming constraints (after bringing all the variables to left-hand side for proper form). Furthermore, the definitions also implies that one or more of these constraints (the one with the smallest right-hand side) will hold with equality.

Therefore, z is simply the largest quantity that satisfies all K of these constraints, which condition is already ensured by maximizing Z = z. Consequently, the equivalent linear programming model is[13]:

$$\text{Maximize } Z = z$$

Subject to

$$\sum_{j=1}^{n} c_{jk}x_j - z \geq 0 \quad \text{for} \quad k = 1,2,\ldots,k$$

$$x_j \geq 0 \quad \text{for} \quad j = 1,2,\ldots,n,$$

and

any other linear programming constraints in the original model.

Second, the nonlinearity which comes from the division operations and and the square root. These nonlinearity factors can be solved by an appropriate procedure.

<u>Note:</u>

The logic in constructing the preceding two models have something in common. They both optimize the allocation of a fixed amount of fund among the activities. This amount of fund covers the initial investments of that activities with no provisions for the operating cost.

The two model can be simply modified by replacing the constraints in Eqs. (4.18), and (4.23) by the following constraints

$$\sum_{(ij)} mqre_{ij} + re_{ij} + \sum_{(ij)} nsrv_{ij} + rv_{ij} = K \qquad (4.24)$$

$$\sum_{(ij)} mqre_{ij} + re_{ij} + \sum_{(ij)} nsrv_{ij} + rv_{ij} \leq K \qquad (4.25)$$

4.2.3 Third Alternative: This alternative is different from the preceding, because it minimizes the cost instead of maximizing the probability, subjects to constraints that warrant the required level of certainty and the amount to be invested within the feasible duration.

Step I: The decision variables are exactly the same as the previous decision variables.

Step II: Two types of constraints will be introduced. The first one which guarantees the required level of certainty, and involves

$$\left[\frac{T - \sum_{(ij)\in C} (\hat{\mu}_{ij})}{\sqrt{\sum (\hat{\sigma}^2_{ij})}} \right]_{p1} \geq Z$$

4.3 SUMMARY

To achieve the objective of maximizing the probability of ending a project before a specified date, the duration and/or the variance should be reduced. Taking this into consideration, three exact models are developed in this chapter. The first two models aim at maximizing the probability of occurrence of the least probable path, while the third aims at minimizing the amount of money invested. For the first two models, two formulations are developed; the first one with the investment cost only while the second one incorporates the operating cost with the investment. The three models are characterized by the nonlinearity, and the large numbers of iterations needed to solve the models.

CHAPTER FIVE

HEURISTIC METHOD

Not all Operation Research mathematical models possess solution algorithms (methods) that always converge to the optimum. There are two reasons for this difficulty [14]:

1. The solution algorithm may be proven to converge to the optimum but only in a theoretical since. Theoretical convergence says that there is a finite upper ceiling on the number of iterations, but it does not say how high this ceiling may be. Thus one can assume hours of computer time without reaching the final iteration. Worse still, if the iterations are stopped prematurely before reaching the optimal, one is usually unable to measure the quality of the obtained solution relative to the true optimum [14].

2. The complexity of the mathematical model may make it impossible to device a solution algorithm. In this case, the model may remain computationally unsolvable [14].

5.1 ALTERNATIVE COMPUTATIONAL METHODS.

The apparent difficulties in mathematical model computations have forced practitioners to seek alternative computational methods. These methods are also iterative in nature, but they do not guarantee optimality. Instead they simply seek a good solution to the problem. Such methods are usually known as heuristics because their logic is based on rules of thumb that are conducive to obtaining a good solution. The advantage of heuristics is that they normally involve less computations when compared with exact algorithms. Also, because they are based on rules of thumb, they normally are easier to explain to users who are not mathematically oriented.

In OR, heuristics are generally employed for two different purposes [14]:

1. They can be used within the context of an exact optimization algorithm to speed up the process of reaching the optimum.

2. They are simply used to a find a good solution to the problem. The resultant solution is not guaranteed to be optimum, and in fact, its quality relative to the true optimum may be difficult to measure.

5.2 JUSTIFICATIONS FOR THE HEURISTIC ADOPTION.

Necessary conditions are existed to justify the use of heuristic method, these reasons or conditions can be summarized by the two following points:

1. The networks problems are combinatorial in nature, and each

combination should be passed by when solving such problems. This entails hours of computer time.

2. The complexity of the mathematical formulation due to the nonlinearity.

5.3 PRELIMINARY INFORMATION.

Before listing the steps or the rules of thumb that will be adopted in the heuristic procedure, two points worth mentioning:

1. As a starting condition, the probabilities of each path in the network should be calculated. To make life easy, a computer program written in Fortran was developed to calculate these probabilities. Optimistic, Most likely, and the pessimistic time estimates for each activity, and the dependencies for the activities (i.e., the predecessors of each activity are the input for the program. Complete documentation for the program can be found in Appendix B.

2. The problem, as was mentioned previously can be stated as follows [10]:

$$\text{Maximize } Z = \Pr[T_n(\{r_{ij}\}) \le \tau\ ]$$

Subject to

$$\sum_{(ij)} r_{ij} \le K$$

$$0 \le r_{ij} \le \overline{r_{ij}}$$

It is easy to translate the objective in the mathematical program of the previous model from a statement on probability to a statement on duration and variance. For it is clear that the durations and the variances to node n equal to the sum of the durations and the variances along each path leading to to node n.

By assumption, an additional investment re > 0 shifts the density function to the left such that the new average duration $\hat{\mu}$ is strictly smaller than the original average duration μ. Also the additional investment rv >0 reduces the new variance such that $\hat{\sigma}^2$ is strictly smaller than the original σ^2. Consequently [10],

$$\Pr[\ \hat{T}_n\ (r) \le t] = F_r(t) > \Pr[\ T_n \le t] = F_\circ\ (t) \qquad (5.1)$$

for all $t \ge t_1$ for some t_1 in the interval $0 \le t_1 \le \mu$.

Furthermore, by the continuity and linearity of $\hat{\mu}$ and $\hat{\sigma}$ in r, it is deduced that $\Pr[\ \hat{T}_n \le t\ ]$ increases with increasing r; That is, the probability, which is better written as $\Pr[\ \hat{T}_n \le t\ |\text{investment} = r\ ]$, is itself continuous and monotone nondecreasing function r at for any fixed t [10].

Hence, maximizing Z is equivalent to minimizing the $E\ [\ \hat{T}_n\ ]$

and $\hat{\sigma}^2_n$, and the new program can be written as

$$\text{Maximize } S = \sum_{(ij)} (\mu_{ij} - \hat{\mu}_{ij}) + \sum_{(ij)} (\sigma_{ij} - \hat{\sigma}_{ij}) \qquad (5.2)$$

Subject to

$$\sum_{(ij)} re_{ij} + \sum_{(ij)} rv_{ij} \leq K$$

$$0 \leq re_{ij} \leq \overline{re}_{ij}$$

$$0 \leq rv_{ij} \leq \overline{rv}_{ij}$$

5.4 HEURISTIC PROCEDURE.

1. Calculate the probability of each path connecting the source node with the sink node. This step can be abolished if previous knowledge of the most critical paths are available (i.e., previous projects of similar nature.)

2. Decide on the level of criticality (i.e., specify the probability under which the probability of a path will be considered critical.)

3. Rank the critical paths in a descending order according to their level of criticality.

4. Maximize the probability of occurrence of the most critical path (i.e, least probable.) according to Eq. (5.2).

The second shortcoming, which is rarely to be occurred, happens when the amount of additional investment is very high and the previous probabilities, with considerable number of paths, are low. This will entail going through many iterations to reach the required solution.

Note:

The logic in constructing the preceding heuristic is similar to the first two alternatives mentioned in Chapter Five. They both optimize the allocation of a fixed amount of fund among the activities. This amount of fund covers the initial investments of that activities with no provisions for the operating cost. The new modified constraints are the same as Eqs. (4.24), and (4.25).

A case study that will clarify the heuristic procedures is found in Chapter Six.

5.6 SUMMARY

To avoid the difficulties associated with the exact models developed in Chapter Four, a heuristic is developed in this chapter, the heuristic is characterized by its nonlinearity and its lesser number of iterations if compared with exact methods. A mechanism for testing the heuristic has not been developed and the quality of the solution is not guaranteed to be optimal.

CHAPTER SIX

CASE STUDY

To illustrate the use of the heuristic approach developed previously, a case study example is considered.

6.1 CASE PROBLEM: LAUNCHING A NEW PRODUCT

It will be assumed that there are no experience with similar products which entails the use of PERT.

6.1.1 Time And Cost Information for Launching the Product:

The activity description, the three time estimates, are viewed in Table 6.1.

Because of the complexity of the problem and the large expected number of the paths, only half of the project will be dealt with.

6.2 PROBLEM SOLUTION

6.2.1 Paths Probabilities Calculations:

Calculating the probabilities of every path is the first step in the heuristic. Of course this needs constructing the PERT network, calculating the expected duration for each activity

Table 6.1. Time Estimates and Activity Description.

Activity description	Optimistic (days)	Most Likely (days)	Pessimistic (days)
Develop product planning specifications	36	60	84
Develop price demand schedule	24	30	36
Conduct market research	42	60	78
Conduct engineering research	49	70	91
Establish distribution outlets	68	80	92
Conduct patent research	7	10	13
Develop laboratory model	66	90	114
Conduct product appraisal	31	40	49
Prepare cost estimates	38	50	62
Design final product	62	80	98
Conduct profit and loss analysis	24	30	36
Determine price	50	65	80

and their respective variance, a knowledge of the maximum expected duration of the project, and the proposed duration for ending the project (i.e., the duration at which the probability is to be maximized). From the interrelationships among the activities, an activity network was developed, the activity network is shown in Figure 6.1 and was developed

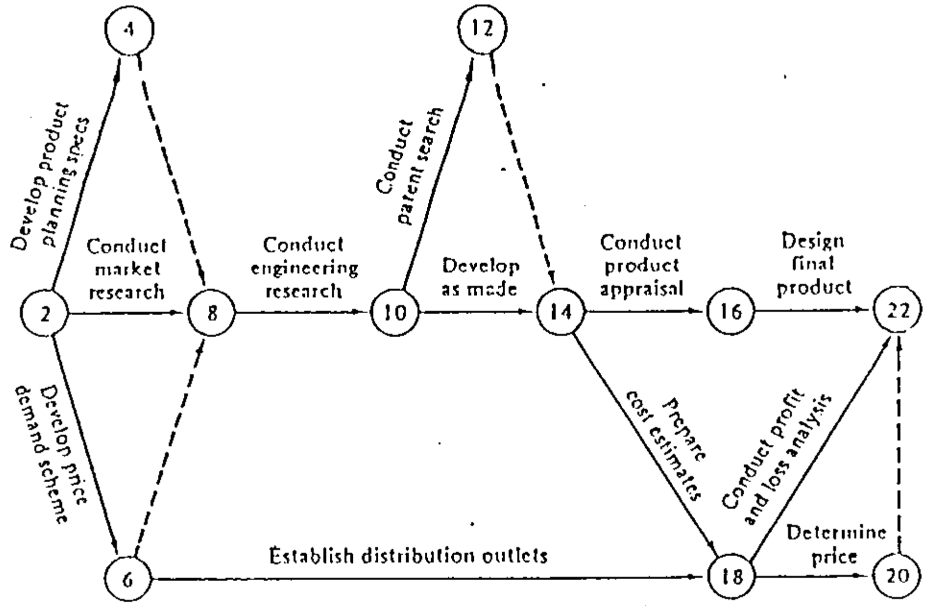

Figure 6.1. Activity Network for "Launching a New Product."

According to Eq. (2.1), the expected duration for the first activity is equal to:

$$t_e = \frac{36 + 4x60 + 84}{6}$$

$$= 60 \text{ days}$$

According to Eq. (2.2) the variance is equal to

$$\sigma_e^2 = \left(\frac{84 - 36}{6}\right)^2$$

$$= 64$$

To simplify the solution, the program in Appendix B, is used to calculate:

1. The expected duration.

2. The variance.

3. Early start, early finish, late start, and the late finish.

4. Total float, free float, intermediate float, and the independent float.

5. All the paths leading from the source node to the sink node.

6. Rank the paths in a descending order according to their level of criticality.

The input for the program is the activities three time estimates, the predecessors of each activity, the required duration and the level of criticality. The outputs of the program that are of concerned are summarized in Table 6.2.

From the output of the program the maximum expected duration was 340 days, so 350 days is chosen as the pilot duration for ending the project.

Providing the program with this duration, all the paths and their respective probabilities were resulted and are shown in Table 6.3.

6.2.2 Choosing the Level of Criticality:

The choice of the level of criticality is a decision made by the top management level in the company organization and depends on many factors among which are, the overall

Table 6.2. Summery of The Program Output.

Activity Number	Expected Value (days)*	Variance (days)^2
1	60	64
2	30	4
3	60	36
4	0	0
5	0	0
6	70	49
7	80	16
8	90	64
9	10	1
10	0	0
11	40	9
12	50	16
13	80	36
14	30	4
15	65	25
16	0	0

* The days with zero expected durations are dummy activities

strategies and polices of the company, the investment rate of return, competition criteria, and the degree of risk the company is willing to take or can withstand.

In this case study a 90% criticality level is chosen. This number is chosen arbitrarily.

6.2.3 Cost Information:

Before proceeding with our problem, the normal and the crash costs for the expected duration and the variance should be identified. Of course these costs estimates are not set hub-hazardly, but need extensive studies and experience. For illustrative purposes, the costs were chosen to be reasonable as much as possible.

Along with our objective, the slope m, and the y intercept m_o for the duration cost curves, with the slope n, and the y intercept n_o for the variance cost curve were calculated.

Also the value of q (the decrease in duration due to one unit increase in the investment) in Eq. (4.8), and the value of s (the decrease in variance due to one unit increase in the investment) in Eq. (4.16).

Table 6.3. All The Paths and Their Respective Probabilties

Path Number	Path Routing	Probability (%)
1	2-7-14	100.000
2	2-7-15-16	100.000
3	3-6-8-11-13	76.115
4	3-6-8-12-15-16	85.993
5	3-6-8-12-14	99.993
6	3-6-9-10-12-15-16	100.000
7	3-6-9-10-12-14	100.000
8	3-6-9-10-11-13	100.000
9	2-4-6-8-11-13	99.916
10	2-4-6-8-12-15-16	99.983
11	2-4-6-8-12-14	100.000
12	2-4-6-9-10-12-15-16	100.000
13	2-4-6-9-10-12-14	100.000
14	2-4-6-9-10-11-13	100.000
15	1-5-6-8-11-13	74.857
16	1-5-6-8-12-15-16	84.375
17	1-5-6-8-12-14	99.982
18	1-5-6-9-10-12-15-16	100.000
19	1-5-6-9-10-12-14	100.000
20	1-5-6-9-10-11-13	100.000

To make things easier Eqs (4.8),(4.9e),(4.16), and (4.17) are rewritten:

$$\hat{\mu} = \mu + qr; \qquad -\mu/r < q < 0 \tag{6.1}$$

$$\hat{\gamma} = m_o + m\mu + mqr + r \tag{6.1a}$$

$$\hat{\sigma}^2 = \sigma^2 + sr \quad ; \qquad -\sigma^2/r < s < 0 \tag{6.2}$$

$$\hat{\beta} = n_o + n\sigma_{ij} + nsr + r \tag{6.3}$$

All the mentioned were calculated and can be viewed in Table 6.4.

6.2.4. The heuristic Iterations:

The number of heuristic iterations depend mainly on the additional amount to be invested. If this amount is large compared with the normal cost, the number of iterations will be increased. Also the number will increase when the number of paths, having probabilities less than the level of criticality, is large.

Two problems will be investigated; the first one will take into account the optimization of the additional investment with no provision to the operating cost. The second will manipulate the problem from the two aspects mentioned lately.

Table 6.4. Problem Parameters

Activity No.	Nd	Cd	Ncd	Ccd	M	M_o	q	Nv	Cv	Ncv	Ccv	n	n_o	s
1	60	40	25,000	48,000	-1000	88000	0.0005	64	37	8000	35000	-1000	42000	0.0006
2	30	10	5,000	30,000	-1250	42500	0.0006	4	0	5000	25000	-5000	25000	0.00015
3	60	20	30,000	125,000	-2375	172500	0.0003	36	16	10,000	30,000	-1000	46000	0.0005
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	70	40	40,000	160,000	-4000	32,000	0.0002	49	25	4,000	16000	-500	28500	0.001
7	80	50	55,000	130,000	-2500	255,000	0.0025	16	9	7000	21000	-2000	39000	0.0004
8	90	50	50,000	154,000	-2600	28,4,000	0.00035	64	16	3000	33000	- 625	43000	0.0006
9	10	10	10,000	10,000	0	0	0	1	0	2000	2800	-800	2800	0.0007
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	40	20	15000	51000	-1800	8,7000	0.00032	9	0	6000	33000	-3000	33000	0.0003
12	50	20	9,000	27,000	-600	39,000	0.00045	16	4	8000	22400	-200	27200	0.00075
13	80	50	41,000	152,000	-3700	337,000	0.0002	36	16	3000	27000	-1200	46200	0.00064
14	30	10	5700	24700	-950	34200	0.00055	4	0	5000	18000	-3250	16000	0.00028
15	65	45	5000	16200	-560	41400	0.0007	25	4	12000	48750	-1750	55750	0.00047

Where

Nd = Normal duration.
Cd = Crash duration.
Ncd = Normal cost for duration.
Ccd = crash cost for duration.
Nv = Normal variance.
Cv = Crash variance.
Ncv = Normal cost for variance.
Ccv = Crash cost for variance.

A) The first problem. According to our level of criticality and referring to Table 6.4, four paths meet the conditions of the heuristic. These paths, ranked in descending order, are:

1. 3-6-8-12-15-16 probability = 85.992 %
2. 1-5-6-8-12-15-16 probability = 84.375 %
3. 3-6-8-11-13 probability = 76.115 %
4. 1-5-6-8-11-13 probability = 74.857 %

Path number four are the most critical path. So the first step is to maximize its probability of occurrence by the additional investment.

Knowing that the total normal cost equals 366700 JD, and the total crash cost equals 1239850 JD, the additional amount to be invested is chosen to be 600000 JD.

A.1 First iteration

4. 1-5-6-8-11-13 probability = 74.857 %

Recalling Eq. (5.2).

$$\text{Maximize } S = \sum_{(ij)} (\mu_{ij} - \hat{\mu}_{ij}) + \sum_{(ij)} (\sigma_{ij} - \hat{\sigma}_{ij})$$

Subject to

$$\sum_{(ij)} re_{ij} + \sum_{(ij)} rv_{ij} \leq K$$

The problem can be written as follows:

$$\text{Maximize } S = q_1re_1 + s_1rv_1 + q_5re_5 + s_5rv_5 + q_6re_6 + s_6rv_6 + q_8re_8 + s_8rv_8 + q_{11}re_{11} + s_{11}rv_{11} + q_{13}re_{13} + s_{13}rv_{13} + q_3re_3 + s_3rv_3$$

Subject to

$re_1 \leq 40000$	$rv_1 \leq 45000$
$re_6 \leq 150000$	$rv_6 \leq 24000$
$re_8 \leq 114285.71$	$rv_8 \leq 60000$
$re_{11} \leq 62500$	$rv_{11} \leq 30000$
$re_{13} \leq 15000$	$rv_{13} \leq 31250$
$re_3 \leq 133333.33$	$rv_3 \leq 40000$

$$e_1 + rv_1 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \leq 313000$$

$$re_3 + rv_3 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \leq 287000$$

$$re_1, rv_1, re_6, rv_6, re_8, rv_8, re_{11}, rv_{11}, re_{13}, rv_{13}, re_3, rv_3 \geq 0$$

After substituting the values of s, q from Table 6.4 in the previous linear formulation, the problem is solved using the QSB package.

The values of the results were as follows:

$re_1 = 40000$	$rv_1 = 45000$
$re_3 = 19000$	$rv_3 = 40000$

$rv_6 = 24000$

$re_8 = 112750$ $rv_8 = 60000$

$rv_{13} = 31250$

Expected duration for path four = 280.5375 days

Variance for path four = 103.0000

Probability = 100 %

Expected duration for path three = 294.8375 days

Variance for path three = 82.000

Probability = 100 %

According to the heuristic approach, paths numbers four and three are no longer critical, and path number two is the critical path now. Next, maximize the probability of occurrence of the three paths.

A.3 Third iteration.

2. 1-5-6-8-12-15-16 probability = 84.375 %
3. 3-6-8-11-13 probability = 76.115 %
4. 1-5-6-8-11-13 probability = 74.857 %

The amount invested on path four

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)} \times (0.9-.74857)$$

= 262194 JD

The amount invested on path three

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)} \times (0.9-.76115)$$

$= 240412$ JD

The amount invested on path two

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)} \times (0.9-.84375)$$

$= 97394$ JD

The problem can be written as follows:

$$\text{Maximize } S = q_1re_1 + s_1rv_1 + q_5re_5 + s_5rv_5 + q_6re_6 + s_6rv_6 + q_8re_8 + s_8rv_8 + q_{11}re_{11} + s_{11}rv_{11} + q_{13}re_{13} + s_{13}rv_{13} + q_3re_3 + s_3rv_3 + q_{12}re_{12} + s_{12}\, rv_{12} + q_{15}re_{15} + s_{15}rv_{15}$$

Subject to

$re_1 \leq 40000$	$rv_1 \leq 45000$
$re_6 \leq 150000$	$rv_6 \leq 24000$
$re_8 \leq 114285.71$	$rv_8 \leq 60000$
$re_{11} \leq 62500$	$rv_{11} \leq 30000$
$re_{13} \leq 15000$	$rv_{13} \leq 31250$
$re_3 \leq 133333.33$	$rv_3 \leq 40000$
$re_{12} \leq 66666.67$	$rv_{12} \leq 16000$
$re_{15} \leq 28571.43$	$rv_{15} \leq 44680.85$

$re_1 + rv_1 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \leq 262194$

$re_3 + rv_3 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \leq 240412$

$re_1 + rv_1 + re_6 + rv_6 + re_8 + rv_8 + re_{12} + rv_{12} + re_{15} + rv_{15} \leq 97394$

$re_1, rv_1, re_6, rv_6, re_8, rv_8, re_{11}, rv_{11}, re_{13}, rv_{13}, re_3,$

$rv_3 + re_{12} + rv_{12} + re_{15} + rv_{15} \geq 0$

After substituting the values of s, q from Table 6.4 in the previous linear formulation, the problem is solved using the QSB package.

The values of the results were as follows:

$rv_1 = 28823$

$re_3 = 52662$ $rv_3 = 40000$

$rv_6 = 24000$

$re_{11} = 62500$ $rv_{11} = 30000$

$rv_{12} = 16000$

$rv_{13} = 31250$

$re_{15} = 28571$

Expected duration for path four = 320.0000 days

Variance for path four = 151.7062

Probability = 99.245 %

Expected duration for path three = 304.2014 days

Variance for path three = 121.0000

Probability = 100 %

Expected duration for path two = 315.0000 days

Variance for path two = 164.7062

Probability = 99.674 %

According to the heuristic approach, paths numbers four, three, and two are no longer critical, and path number one is the critical path now. Next, maximize the probability of occurrence of all the four paths.

A.4 Fourth iteration.

1. 3-6-8-12-15-16 probability = 85.993 %
2. 1-5-6-8-12-15-16 probability = 84.375 %
3. 3-6-8-11-13 probability = 76.115 %
4. 1-5-6-8-11-13 probability = 74.857 %

The amount invested on path four

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)+(.9-.85993)} \times (0.9-.74857)$$

= 235018 JD

The amount invested on path three

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)+(.9-.85993)} \times (0.9-.76115)$$

= 214355 JD

The amount invested on path two

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)+(.9-.85993)} \times (0.9-.84375)$$

= 86850 JD

The amount invested on path one

$$= \frac{600000}{(.9-.74857)+(.9-.76115)+(.9-84375)+(.9-.85993)} \times (0.9-.85993)$$

= 63747 JD

The problem can be written as follows:

$$\text{Maximize } S = q_1 re_1 + s_1 rv_1 + q_5 re_5 + s_5 rv_5 + q_6 re_6 + s_6 rv_6 + q_8 re_8 + s_8 rv_8 + q_{11} re_{11} + s_{11} rv_{11} + q_{13} re_{13} + s_{13} rv_{13} + q_3 re_3 + s_3 rv_3 + q_{12} re_{12} + s_{12} rv_{12} + q_{15} re_{15} + s_{15} rv_{15}$$

Subject to

$re_1 \le 40000$	$rv_1 \le 45000$
$re_6 \le 150000$	$rv_6 \le 24000$
$re_8 \le 114285.71$	$rv_8 \le 60000$
$re_{11} \le 62500$	$rv_{11} \le 30000$
$re_{13} \le 15000$	$rv_{13} \le 31250$
$re_3 \le 133333.33$	$rv_3 \le 40000$
$re_{12} \le 66666.67$	$rv_{12} \le 16000$
$re_{15} \le 28571.43$	$rv_{15} \le 44680.85$

$$re_1 + rv_1 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \le 235018$$

$$re_3 + rv_3 + re_6 + rv_6 + re_8 + rv_8 + re_{11} + rv_{11} + re_{13} + rv_{13} \le 214385$$

$$re_1 + rv_1 + re_6 + rv_6 + re_8 + rv_8 + re_{12} + rv_{12} + re_{15} + rv_{15} \le 86850$$

$$re_3 + rv_3 + re_6 + rv_6 + re_8 + rv_8 + re_{12} + rv_{12} + re_{15} + rv_{15} \le 63747$$

$$re_1,\ rv_1,\ re_6,\ rv_6,\ re_8,\ rv_8,\ re_{11},\ rv_{11},\ re_{13},\ rv_{13},\ re_3,\ rv_3 + re_{12} + rv_{12} + re_{15} + rv_{15} \ge 0$$

After substituting the values of s, q from Table 6.4 in the previous linear formulation, the problem is solved using the QSB package.

The values of the results were as follows:

$re_1 = 17850$	$rv_1 = 45000$

$rv_3 = 39750$

$rv_6 = 24000$

$re_{11} = 62500$ $rv_{11} = 30000$

$re_{13} = 24418$ $rv_{13} = 31250$

Expected duration for path four = 306.1914 days

Variance for path four = 142.0000

Probability = 99.986 %

Expected duration for path three = 315.1164 days

Variance for path three = 121.1250

Probability = 99.921 %

Expected duration for path two = 326.0750 days

Variance for path two = 167.0000

Probability = 96.784 %

Expected duration for path one = 335.0000 days

Variance for path two = 146.1250

Probability = 89.251 %

The heuristic is stopped now because all the critical paths were maximized, and the resultant critical path was path number one.

So the probability of ending the project in less than or equal to 350 days is 89.251 % which is very much near the our level of criticality. The thing that worth mentioning is that this solution is not guaranteed to be optimal, and the optimal solution could be far from this solution.

B) Second problem. In this problem, the operating cost will be considered. The analysis are exactly the same as the previous problem. The only difference is in the constraints that are related with the path.

In this problem the constraints related to the paths are set as follows:

The amount invested on each activity constructing the path and its operating cost should be less than the total amount to be invested. Recalling Eqs. (6.1a), and (6.3)

$$\hat{\gamma} = m_o + m\mu + mqr + r$$

$$\hat{\beta} = n_o + n\sigma^2 + nsr + r$$

The additional cost resulting from the investment for each activity is:

$$= m \times q \times re + re + n \times s \times rv + rv$$

Concerning the problem, the constraints for each path at the fourth iteration will be as follows:

Path number four:

$1.5re_1 + 1.6rv_1 + 1.8re_6 + 1.5rv_6 + 1.91re_8 + 1.5rv_8 + 1.576re_{11}$
$+ 1.9rv_{11} + 1.74re_{13} + 1.768rv_{13} \leq 235018$

Path number three:

$1.7125re_3 + 1.5rv_3 + 1.8re_6 + 1.5rv_6 + 1.91re_8 + 1.5rv_8 + 1.576re_{11}$
$+ 1.9rv_{11} + 1.74re_{13} + 1.768rv_{13} \leq 214385$

Path number two:

$1.5re_1 + 1.6rv_1 + 1.8re_6 + 1.5rv_6 + 1.91re_8 + 1.5rv_8 + 1.27re_{12}$
$+ 1.9rv_{12} + 1.392re_{15} + 1.8225rv_{15} \leq 86850$

Path number one:

$1.7125re_3 + 1.5rv_3 + 1.8re_6 + 1.5rv_6 + 1.91re_8 + 1.5rv_8 + 1.27re_{12}$
$+ 1.9rv_{12} + 1.392re_{15} + 1.8225rv_{15} \leq 86850$

The results of the problem are as follows:

B.1 First iteration.

$re_1 = 40000$ $rv_1 = 45000$

$rv_6 = 24000$

$re_8 = 98560$ $rv_8 = 60000$

$re_{11} = 625000$

$rv_{13} = 31250$

Expected duration = 85.50401 days

Variance =103.0000

Probability = 100 %

B. 2 Second iteration

re_1 = 40000

re_3 = 26861

rv_1 = 45000

rv_3 = 40000

rv_6 = 24000

rv_8 = 60000

rv_{13} = 31109

Expected duration for path four = 320.0000 days

Variance for path four = 103.0902

Probability = 99.841 %

Expected duration for path three = 331.9417 days

Variance for path three = 82.0902

Probability = 97.67 %

B. 3 Third iteration.

rv_3 = 40000

rv_6 = 24000

re_{11} = 56575

rv_{12} = 11381

rv_{13} = 31250

re_{15} = 28571

Expected duration for path four = 321.8960 days

Variance for path four = 178.0000

Probability = 98.214 %

Expected duration for path three = 321.8960 days

Variance for path three = 130.0000

Probability = 99.305 %

Expected duration for path two = 315.0000 days

Variance for path two = 185.4642

Probability = 99.492 %

B.4 Forth iteration

$rv_1 = 45000$

$rv_3 = 32600$

$rv_6 = 6179$

$re_{11} = 62500$

$rv_{13} = 31250$

$re_{15} = 4010$

Expected duration for path four = 320.0000 days

Variance for path four = 168.8210

Probability = 98.928 %

Expected duration for path three = 320.0000 days

Variance for path three = 151.5210

Probability = 99.245 %

Expected duration for path two = 332.1930 days

Variance for path two = 184.8210

Probability = 90.320 %

Expected duration for path one = 332.1930 days

Variance for path two = 167.5210

Probability = 91.465 %

According to this problem, path number two is the critical path with probability of occurrence of 90.32 %

6.3 CRITERIA FOR DESIGNATING FUNDS ON CRITICAL PATH

In the previous case study, the proximity of the critical paths to the criticality level criterion was used. In this section two other criteria will be used:

1- Designating the same amount of money for each critical path.

2- Omitting the constraints related to the critical paths.

To compare the results of the three criteria, the probabilities of iteration four for the second problem are calculated and summarized in Table 6.5.

Table 6.5. Comparison of The Results for The Different Criteria for The Second Problem.

Criteria	Probability
First	90 320%
Second	78.523%
Third	86.433%

6.4 SUMMARY

In this section the steps that should be followed in solving a typical problem and the results of the case study are discussed.

To start solving any problem, the following steps are to be followed:

1- Identify the activities: Identifying the activities entail the studying of all the alternatives associated with these activities. This, of course, depends on the risk analysis discussed in chapter three.

2- Build the interrelation between activities: This step is important in order to construct the activity network.

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

In this research the scarcity or even the absence of subjects dealing with the crashing in PERT networks were the major obstacles that were faced hence, the subjects that were discussed in literature review (chapter two) were lacking previous works on this subject. The topics were merely basic concepts that started with a brief review of the evolution of CPM, PERT and SANs, this was followed by comparisons between networking methods after which, a comprehensive study of PERT was presented with emphasis on the assumptions used in the analysis and solution in the PERT network. The concept of crashing was then introduced followed by the different methods used to solve mathematical formulation. The last sections were on the optimal Time-cost trade-off in GERT networks.

Project initialization needs activities identification, for each activity, several alternatives might be available. The choice of the best alternative needs evaluation of alternatives under risk and uncertainty conditions. For this purpose a separate chapter dealing with risk and uncertainties was devoted. The chapter included many topics among which are; the causes of risk and uncertainty, decision criteria and methods for risk analysis, and risk interpretation for PERT networks.

In chapter four, Three exact models were developed; the first two models aimed at maximizing the probability of occurrence of the least probable path while the third model aimed at minimizing the amount of money invested. Among the assumptions was the linearity of the operating and investment cost function. This approximation simplifies the development of the model but on the account of the practicality of the problem since each problem has its own cost function. For the first two models, two formulations were developed; the first one with the investment cost only while the second one incorporated the operating cost with the investment. The nonlinearity and the large numbers of iterations that will be needed to solve the models were the common features for the three models.

A heuristic was developed in chapter five to avoid the above mentioned difficulties, it is nonlinear with less number of iterations if compared with exact models. To maximize the efficiency of the heuristic, the heuristic could be used in two forms, according to the amount invested:

1. If the amount to be invested is small relative to the initial investment, it is recommended to follow the heuristic as it is, in other words, start maximizing the least critical path and keep on adding paths until a stopping condition is encountered

2. It the amount to be invested is large compared with the initial investment, it is recommended to start the heuristic from the end, in other words, start maximizing all the critical paths and keep on subtracting path by path until a stopping condition is encountered.

Following these steps, the number of iterations needed to encounter a stopping condition, will be reduced. To facilitate and reduce the time of solution, a computer program written in fortran language was developed. The first three steps of the heuristic are among the output of the program.

In chapter six, a theoretical case study was introduced, to test the developed program and to support the validity of the heuristic. Two problems were solved; one with the investment cost only and the other with the operating cost. According to the heuristic a fixed amount of money should be allocated for each path. In the case study three criteria were used; The proximity to the criticality level, the same amount for each path, and the cancellation of the constraints dealing with this aspect. The results were according to expectations and the, proximity to the criticality level showed better results.

Followings, are future research areas:

1. Working on the true distribution of the activity duration.
2. Finding a better estimate for the expected duration and the variance.

3. Using the actual nonlinear cost cost curves for time cost trade off

Possible extensions:

1. Developing the concept of this thesis by working more on developing criteria for designating the amount invested on each path.

2. Developing a mechanism for testing and assessing of the heuristic.

3. Developing an integrated program that is able to solve the PERT network, find all the paths, and optimize the probability of meeting certain schedule.

Appendix A
Areas Under the Normal Curve

Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

```
     *GRAM',6(/),20X,'PRESS ANY KEY TO ENTER')
      READ(*,301)MM
      IF(MM.NE.Q)GO TO 3674
3674  DO 107 I=1,400
107   ZZ(I)= 0.0
      WRITE(*,201)
201   FORMAT(4(/),5X,'DO YOU HAVE AN ALREADY EXISTING DATA BAS
     *E FILE [Y/N]',$)
      READ(*,301)MM
301   FORMAT(A1)
      IF(MM.EQ.'Y'.OR.MM.EQ.'y')GO TO 901

      WRITE(*,401)
401   FORMAT(5(/))
      DO 544 I=1,400
      WRITE(*,533)I
533   FORMAT(1(/),6X,'================= NORMAL VALUES (',I3,'
     *) =',$)
544   READ(*,601)ZZ(I)
601   FORMAT(F11.5)
      WRITE(*,701)

701   FORMAT(1(/),5X,'WHAT DO YOU WANT TO CALL THIS INPUT DATA
     *FILE ? PLEASE ENTER YOUR FILE NAME WITH NO MORE THAN 12
     *CHARACTERS, PREFERABLY IN THE FORM INDATA[].DAT, WHERE A
     * VERSION NUMBER CAN BE INSERTED INSIDE THE BRACKETS:',$)
      READ(*,'(A)')FN
      OPEN(2,FILE=FN,STATUS='NEW')
      DO 801 I=1,400
801   WRITE(2,601)ZZ(I)
      CLOSE(2)
      GO TO 102

901   WRITE(*,'(A)')'***** WHAT IS THE NAME OF YOUR INPUT DATA
     * FILE ?'
```

```
      READ(*,'(A)')FN
      OPEN(2,FILE=FN)
      DO 202 I=1,400
202   READ(2,601)ZZ(I)
      CLOSE(2)

102   WRITE(*,29)
 29   FORMAT(1(/),5X,'THERE IS A SET OF INPUT DATA FILES CORRE
     *SPONDING TO DIFFERENT SYSTEMS .',1(/),5X,'DOES YOUR SYST
     *EM HAVE AN ALREADY EXISTING INPUT DATA FILE ? [Y/N] ',$)
      READ(*,6961)AF
6961  format(A1)
      IF(AF.EQ.'Y'.OR.AF.EQ.'y') GO TO 4

  2   WRITE(*,5)
  5   FORMAT(1(/),16X,'ENTER YOUR MAX. # OF ACTIVITIES N ?[2]
     * ',$)
      READ(*,59)N
 59   FORMAT(I2)
      IF(N.GT.60) THEN
      WRITE(*,16)
 16   FORMAT(2(//),10X,'MAX.# IS EXCEEDED,PRESS Ctrl AND C, MO
     *DIFY THE DIMENSION OF B(I,J) IF YOU WANT ')
      GO TO 2
      ENDIF
      WRITE(*,4732)
4732  FORMAT(2(/),5X,'WOULD YOU LIKE TO CHANGE THE MAX. NUMBER
     * OF ACTIVITIES [Y/N]',$)
      READ(*,301)MM
      IF(MM.EQ.'Y'.OR.MM.EQ.'y')GO TO 2

C***  NOW ENTER THE INTERCONNECTED MATRIX OF THE NETWORK B(I,J)
C***  THE MATRIX DIM. SHOULD BE AT MOST 60*7

      DO 28 I=1,60
```

```
      DO 28 J=2,7
28    B(I,J)=-1

      DO 23 I=1,N
23    B(I,1)= I

C***  YOU ARE GOING TO READ THE INTERCONNECTED MATRIX

 812  WRITE(*,618)
 618  FORMAT(1(/),21X,'[ NOW ENTER YOUR CONNECTED MATRIX ] ?',
     *1(/))
      DO 35 I=1,N
      WRITE(*,30)I
  30  FORMAT(15X,'WHAT IS THE # OF PREDECESSORS OF ACTIVITY :
     *',I2,' ? ',$)
      READ(*,59)M
      B(I,7)=M
      DO 71 J=2,M+1
      WRITE(*,6)I,J
      READ (*,59)B(I,J)
  71  CONTINUE
   6  FORMAT(15X,'B(',I2,',',I2,') ? ',$)
  35  CONTINUE

C***  HERE YOU ARE GOING TO SPECIFY THE END POINTS

3923  WRITE(*,93)
  93  FORMAT(2(/),15X,'[ ENTER THE # OF END POINTS:? ]',$)
      READ(*,59)NLS(1)
      DO 930 I=1,NLS(1)
      WRITE(*,97)I
  97  FORMAT(15X,'END(',I2,')? ',$)
      READ(*,59)LPN(I)
 930  CONTINUE
      WRITE(*,7891)
```

```
7891  FORMAT(2(/),5X,'WOULD YOU LIKE TO CHANGE ANY OF YOUR ACT
     *IVITY NETWORK PREDECESSORS [Y/N]',$)
      READ(*,301)MM
      IF(MM.EQ.'Y'.OR.MM.EQ.'y')GO TO 812

599   WRITE(*,253)
253   FORMAT(24(/),15X,'[ NOW ENTER ACTIVITY DURATIONS, REAL F
     *IELD ]',$)
      DO 10 I=1,N
      WRITE(*,11)I
11    FORMAT(1(//),5X,'THE THREE TIME ESTIMATES OF ACTIVITY ',
     *I2,'ARE:= ')
      WRITE(*,333)
333   FORMAT(1(/),3X,'ENTER THE OPTIMISTIC TIME')
      READ(*,82)X
      WRITE(*,222)
222   FORMAT(1(/),3X,'ENTER THE MOST LIKELY TIME')
      READ(*,82)WW
      WRITE(*,111)
111   FORMAT(1(/),3X,'ENTER THE PESSIMISTIC TIME')
      READ(*,82)Z
82    FORMAT(F5.2)
      D(I)=(X+4*WW+Z)/6.0
      CC(I)=D(I)
      V(I)=((Z-X)/6.0)**2
      VV(I)=V(I)
10    CONTINUE
      WRITE(*,587)
587   FORMAT(2(/),5X,'WOULD YOU LIKE TO CHANGE YOUR ACTIVITIES
     *  DURATION  [Y/N]',$)
      READ(*,301)MM
      IF(MM.EQ.'Y'.OR.MM.EQ.'y')GO TO 599

352   WRITE(*,9754)
9754  FORMAT(24(/),5X,'ENTER THE REQUIRED DURATION')

      WRITE(1,15)
      DO 1479 I=1,N
      WRITE(1,82)CC(I)
1479  CONTINUE
```

```
      WRITE(1,15)
      DO 1389 I=1,N
      WRITE(1,*)VV(I)
1389  CONTINUE
      WRITE(1,15)
      WRITE(1,*)Q
      WRITE(1,15)
64    WRITE(1,59)NLS(1)
      DO 83 I=1,NLS(1)
      WRITE(1,91)LPN(I)
 91   FORMAT(5X,I3)
 83   CONTINUE
      CLOSE(1)
      go to 3

  4   WRITE(*,'(A)')'***** WHAT IS THE NAME OF YOUR INPUT DATA
     * FILE ?'
      READ(*,'(A)')FN
      OPEN(1,FILE=FN)
      READ(1,59)N
      DO 250 I=1,N
      DO 260 J=1,7
      READ(1,52)B(I,J)
 260  CONTINUE
      READ(1,15)
 250  CONTINUE
      READ(1,15)
      READ(1,15)
      READ(1,15)
      DO 6983 I=1,N
      READ(1,82)D(I)
6983  CONTINUE
      READ(1,15)
      DO 2369 I=1,N
      READ(1,82)CC(I)
2369  CONTINUE
```

```
937     FORMAT(I2)
        DO 9412 J=2,7
        B(N1,J)=-1
9412    CONTINUE
        WRITE(*,30)N1
        READ(*,59)M1
        B(N1,7)=M1
        DO 284 R=2,M1+1
        WRITE(*,6)N1,R
        READ(*,59)B(N1,R)
284     CONTINUE
323     CONTINUE
        ENDIF
        GO TO 627

666     WRITE(*,496)
        READ(*,59)N5
        DO 857 I=1,N5
        WRITE (*,936)I
        READ(*,937)N2
        WRITE(*,11)N2
        WRITE(*,333)
        READ(*,82)X1
        WRITE(*,222)
        READ(*,82)X2
        WRITE(*,111)
        READ(*,82)X3
        D(N2)=(X1+4*X2+X3)/6
        CC(N2)=D(N2)
        V(N2)=((X3-X1)/6)**2
        VV(N2)=V(N2)
857     CONTINUE
        GO TO 627

586     DO 143 I=1,N
        K=3
```

```
      AA(I,1)=I
      AA(I,2)=0
      DO 629 M=1,N
      DO 763 J=2,6
      IF(B(M,J).NE.I)GO TO 763
      AA(I,2)=AA(I,2)+1
      AA(I,K)=B(M,1)
      K=K+1
763   CONTINUE
629   CONTINUE
143   CONTINUE


C**** CALCULATIONS OF EARLY STARTS
      DO 56 I=1,N
      DO 57 J=1,6
   57 K4(J)=0.0
      IF(B(I,7).EQ.0)THEN
      ES(I)=0.0
      ELSE
      IF(B(I,7).EQ.1)THEN
      ES(I)=CC(B(I,2))+ES(B(I,2))
      ELSE
      IF(B(I,7).GE.2)THEN
      KO=B(I,7)
      DO  58 J=1,KO
      K4(J)=CC(B(I,J+1))+ES(B(I,J+1))
   58 CONTINUE
      AMAX1=K4(1)
      DO 519 J=2,KO
      IF(AMAX1.LT.K4(J))  AMAX1=K4(J)
  519 CONTINUE
      ES(I)=AMAX1
      ENDIF
      ENDIF
      ENDIF
```

```
   56   CONTINUE

C**** CALCULATIONS OF LATE FINISHS

        ILS=0
        DO 9012 IJ=1,6
 9012   K2(IJ)=0.0
        DO 60 I=1,N
        IF(AA(I,2).EQ.0)THEN
        ILS=ILS+1
        K2(ILS)=ES(AA(I,1))+CC(AA(I,1))
        ENDIF
   60   CONTINUE

        AMAX2=K2(1)
        DO 61 I=2,ILS
        IF(AMAX2.LT.K2(I)) AMAX2=K2(I)
   61   CONTINUE

        DO 9145 J=1,6
 9145   K3(J)=0.0

        DO 62 I=1,N
         J=N-I+1
        IF(AA(J,2).EQ.0) THEN
        LF(J)=AMAX2
         ELSE
        IF(AA(J,2).EQ.1) THEN
        LF(J)=LF(AA(J,3))-CC(AA(J,3))
         ELSE
        IF(AA(J,2).GE.2) THEN
        K8=AA(J,2)
        DO 66 L1=1,K8
        K3(L1)=LF(AA(J,L1+2))-CC(AA(J,L1+2))
   66   CONTINUE
```

```
      AMIN1=K3(1)
      DO 63 M=2,K8
      IF(AMIN1.GT.K3(M)) AMIN1=K3(M)
   63 CONTINUE
      LF(J)=AMIN1
      ENDIF
      ENDIF
      ENDIF
   62 CONTINUE

C**** HERE ARE THE CALCULATIONS

      DO 14 I=1,N

      EF(I)=ES(I)+CC(I)
      LS(I)=LF(I)-CC(I)
      TF(I)=LF(I)-EF(I)
      IF(AA(I,3).EQ.0) ES(AA(I,3))=LF(I)
      FF(I)=ES(AA(I,3))-EF(I)
      IDF(I)=(ES(AA(I,3))-LF(B(I,2)))-CC(I)
      ITF(I)=TF(I)-FF(I)

   14 CONTINUE




      DO 75 K=1,15
   75 LOC(K,2)=1
      K=1
      MNB=LOC(K,2)
      DO 101 I=1,N
      DO 90 J=3,15
      M=J
      IF(B(I,J).EQ.-1) GO TO 95
      LOC(K,2)=LOC(K,2)+1
```

```
      LOC(K,1)=I
   90 CONTINUE
   95 IF(M.GE.4) K=K+1
      IF(LOC(K,2).GT.MNB) MNB=LOC(K,2)
  101 CONTINUE

C***  PRINTING THE INTERCONNECTED MATRIX

      WRITE(3,604)
  604 FORMAT(2(/),5X,'THE INTERCONNECTED MATRIX IS :',1(/),5X
     *,30('-'),1(/))
      MNL=K-1
      DO 110 I=1,N
      DO 115 J=1,7
      WRITE(3,51) B(I,J)
 51   FORMAT(2X,I3,$)
  115 CONTINUE
      WRITE(3,15)
 15   format(1x)
  110 CONTINUE


C**** HERE IS PRINTING OF OUTPUT RESULTS


3856  WRITE(3,21)
  21  FORMAT(5(/),29X,'THE OUTPUT RESULTS ARE :',1(/),29X,24('
     *-'),1(/))
      WRITE(3,1576)
1576  FORMAT(3X,'ACTIVITY',3X,'DURATION',3X,'EARLY START',3X,'
     *EARLY FINISH',3X,'LATE START',3X,'LATE FINISH',1(/),3X,8
     *('-'),3X,8('-'),3X,11('-'),3X,12('-'),3X,10('-'),3X,11('
     *-'))
      DO 1623 I=1,N
      WRITE(3,17)I,CC(I),ES(I),EF(I),LS(I),LF(I)
```

```
   17  FORMAT(5X,I2,6X,f6.1,7X,f6.1,9X,f6.1,8X,f6.1,8X,f6.1)
 1623  CONTINUE

       WRITE(*,4912)
 4912  FORMAT(1(//),3X,'ENTER YOUR CRITICALITY LEVEL  ',$)
       READ(*,3691)WX
 3691  FORMAT(F6.5)

       WRITE(3,18)
   18  FORMAT(2(/),3X,'ACTIVITY',3X,'TOTAL FLOAT',3X,'FREE FLOA
      *T',3X,'INTER.FLOAT',3X,'INDP. FLOAT',1(/),3X,8('-'),3X,1
      *1('-'),3X,10('-'),3X,12('-'),3X,11('-'))
       DO 19 I=1,N
       WRITE(3,20)I,TF(I),FF(I),ITF(I),IDF(I)
   20  FORMAT(5X,I2,8X,f6.1,7X,f6.1,8X,f6.1,8X,f6.1)
   19  CONTINUE

       DO 9000 IL=1,NLS(1)
       CALL PATH(PP,FP,LB,LLB,KA,LPN,K1,N,NP,LOC,B,MNL,MNB,KX,I
      *Z,IA,II,KLS,IL,KQ,CC,VV,Q,ZZ,W1,W2,W3,W4,WX,MAX)
 9000  continue



       WRITE(*,977)
  977  FORMAT(3(/),3X,'CHOOSE ONE OF THE FOLLOWING:',2(/),' 1.
      *ENTER NEW PROBLEM',1(/),' 2. QUIT TO DOS')
       READ(*,698)JZ
       IF(JZ.EQ.1)GO TO 102
       IF(JZ.EQ.2)GO TO 9239


 9239  STOP
       END
```

```
C                     BEGINING OF PATH SUBROUTINE

      SUBROUTINE PATH(PP,FP,LB,LLB,KA,LPN,K1,N,NP,LOC,B,MNL,MN
     *B,KX,IZ,IA,II,KLS,IL,KQ,CC,VV,Q,ZZ,W1,W2,W3,W4,WX,MAX)
      REAL CC(60),VV(60),ZZ(400),W2(20),W4(20),MAX
      INTEGER PP(1025,25),FP(20,25),LOC(20,2),LPN(5),KQ,LOA,W1
     *(20),K1(20),KA(20),LB(1025),LLB(1025),B(60,7),R,H,W3(20)


      NP=1
      DO 111 J=1,15
      NP=NP*LOC(J,2)
 111  CONTINUE

      DO 11 I=1,NP
      DO 11 J=1,25
  11  PP(I,J)=-1

      DO 13 I=1,150
      DO 13 J=1,25
  13  FP(I,J)=-1

      DO 719 I=1,15
      KA(I)=-1
      K1(I)=0
 719  CONTINUE

      KX=LPN(IL)

      DO 1000 R=1,NP
      PP(R,1)=LPN(IL)

      IF(LOC(15,2).GT.1) THEN
      DO 72 J=1,LOC(15,2)-1
```

```
      II=LOC(15,1)
      JJ=J+1
      KK=J+2

      IA=B(II,JJ)
      B(II,JJ)=B(II,KK)
      B(II,KK)=IA

72    CONTINUE
      K1(1)=K1(1)+1
      ELSE
      KA(1)=0
      ENDIF

600   IF(K1(1).EQ.LOC(15,2)+1.OR.KA(1).EQ.0) THEN
      IF(LOC(14,2).GT.1) THEN
      DO 77 J=1,LOC(14,2)-1
      II=LOC(14,1)
      JJ=J+1
      KK=J+2

      IA=B(II,JJ)
      B(II,JJ)=B(II,KK)
      B(II,KK)=IA

77    CONTINUE
      K1(2)=K1(2)+1
      ELSE
      KA(2)=0
      ENDIF
      ENDIF

      IZ=LOC(14,2)
      IF(KA(1).EQ.0) IZ=IZ+1
      DO 839 KIS=3,15
      IRF=16-KIS


      DO 814 MOS=2,14
```

```
      ELSE
      IF(LOA.LT.-400)THEN
      P1=0.0
      ELSE
      IF(LOA.LT.0)THEN
      P1=1.-ZZ(ABS(LOA))
      ELSE
      P1=ZZ(LOA)
      ENDIF
      ENDIF
      ENDIF
      IF(P1.LE.WX)THEN
      IS=IS+1
      W1(IS)=KQ
      W2(IS)=P1
      ENDIF
      WRITE(3,321)P1,YV,VY
 321  FORMAT(1(//),3X,'THE PROBABILITY OF OCCURRENCE IS ',F9.5
     *,'  D = ',
     *F10.5,'    V = ', F10.5,$)
      WRITE(3,15)
 15   FORMAT(1X)
      ENDIF
      ENDIF
140   CONTINUE
      WRITE(3,669)KQ
669   FORMAT(3(/),3X,'THE NUMBER OF THE PATHS IS :','(',I3,' )
     *')
      DO 1592 I=1,IS
      MAX=W2(1)
      K=1
      DO 4169 J=2,IS
      IF(MAX.GT.W2(J))GO TO 4169
      MAX=W2(J)
      K=J
```

```
4169  CONTINUE
      W3(I)=W1(K)
      W2(K)=0
      W4(I)=MAX
1592  CONTINUE
      WRITE(3,9128)
9128  FORMAT(4(/),3X,'THE CRITICAL PATHS RANKED IN DESCENDING
     *ORDER ARE',$)
      DO 1180 I=1,IS
      WRITE(3,3982)W3(I),W4(I)
3982  FORMAT(2(/),2X,'CP. ',I2,3X,F9.5,$)
1180  CONTINUE
      RETURN
      END
C                         END OF PATH SUBROUTINE
```

REFERENCES

[1] Robert W. Miller, "Schedule, Cost, and Profit Control with PERT" : A Comprehensive Guide for Program, New York: NY Mc Graw Hill, 1963.

[2] Battersby, A., "Network Analysis for Planning and Scheduling," London, Mac Millan and Co., Ltd. 1987.

[3] Avramidis N.A., Kennth W.B., and James R.W., "Simulation of Stochastic Activity Networks Using Path Control Variates," Naval Research Logistics, Vol. 38, No. 2, April, 1991, pp 183-201.

[4] Antoine E.N., "Networking Methods for Project Planning and Control," Journal of The Construction Division, ASCE, Vol. 100, No. CO3, September, 1974, pp 123-138.

[5] Calin p., and John D.B., "Development in CPM, PERT, and Network Analysis,"Journal of The Construction division, ASCE, Vol. 101, No. CO4, December, 1975, pp 203-218.

[6] Ravindra K.A., and James B.O., "Distance-Directed Augmenting Path Algorithms for Maximum Flow and Parametric Maximum Flow Problems," Naval Research Logistics, Vol. 38, No. 3, June, 1991, pp 413-430.

[7] Gary E. Whitehouse, "Systems Analysis and Design using Network Techniques," Englewood Cliffs: NJ, Prentice Hall, 1973.

[8] Joseph J. Miller, Cecil R. Phillips, "Project Management with CPM and PERT," New York: NY, Reenhold Publishing corporation, 1977.

[9] Robert B. Harris, "Precedence and Arrow Networking Techniques," New York: NY, John Wiley & Sons, 1968.

[10] Salah E. Elmaghraby, "Activity Networks: Project Planning and Control by Networks Models," New York, NY, John Wiley & Sons, 1977.

[11] John R. Canada, "Capital Investment Decision Analysis for Management and Engineering,"Englewood Cliffs: NJ, Prentice Hall, 1980.

[12] Eugene L. Grant, W. Grant Ireson, and Richard S. Leavenworth, "Principles of Engineering Economy," New York, NY, John Wiley & Sons, 1982.

[13] Frederick S.H., and Gerald J.L., "Introduction to Operation Research, Oakland, California, Holden-Day, Inc, 1980.

[14] Hamdy A. Taha, "Operation Research," New York, NY, MacMillan Publishing Company, 1982.